On the quantum hydrodynamics of Newtonian fluids

by E. BUFFONI
Hydraulics Laboratory
Department of Civil Engineering,
University of Pisa v. Gabba, 22, 56126 Pisa, Italy.

Submitted to Proceedings of the Royal Society, London

Summary

To interpret the experimental results obtained by wide-ranging research on vortex shedding by a cylinder, a formulation is proposed of a quantum heory that is valid for all Newtonian fluids. The discovery has, in fact, been made of a constant that is valid for all fluids, depending on kinematic viscosity, which has important applications in the hydrodynamic field, comprising transition to turbulence, vortex shedding under critical and subcritical conditions, and some of the phenomena present in the viscous boundary layer.

Introduction

The study of turbulence still includes unsolved problems. Despite this, the introduction of supercomputers will make it possible to improve our knowledge, starting with the direct numerical integration of basic equations. At present, we can interpret the experimental data using models and theories that are called phenomenological, even if these too are based on the fundamental principles of dynamics and physics. Typical examples of this are the wall law of Prandtl and Von Karman, which depends on an analogy with the kinetic theory for gases. This law, which corresponds very closely to experimental data, has been successfully applied throughout the last century to various fields within hydraulics and fluid mechanics. Starting in the early 1980s, the present author had an intuitive conviction that certain features of turbulence follow a quantum pattern; this hypothesis had first been put forward by Levi (1983), but unfortunately that formulation comprised notable inner contradictions, so that it could not be applied; a quantum theory, in fact, requires a fundamental constant whose value must be determined experimentally. Working at the Institute of Hydraulics, University of Pisa, the present author has therefore undertaken a broad experimental investigation on vortex shedding which has led to the discovery of that constant. As is well known, the simplest source of vorticity found in nature is a uniform flow about a circular cylinder at supercritical Reynolds numbers. This cylinder will then shed a double wake of vortices, and an instrument placed downstream will record an almost sinusoidal signal whose spectrum contains a single band with a Strouhal frequency. This topic has been studied by various authors, among whom we will recall only the most important: Roshko (1954), Tritton (1971) and Friehe (1980).

However, these papers do not make clear the triggering mechanism of this phenomenon once Reynolds's critical number, which is about 50, has been reached. During the last few years, therefore, an extensive experimental enquiry has been carried out, and its results have already been published (Buffoni 1993, 1995, 1996, 1997). These results, not from a wish for originality, but to reflect the experimental facts, have been developed into a coherent theory, which possesses a considerable forecasting capability; in this way the phenomenon itself, which had been rather difficult to understand, turns out to be simple.

Experimental Apparatus

A device able to support cylinders of various diameters was inserted in an open channel with walls of glass, 9 m long and 50 cm wide, set up in Laboratory No.1 of the Hydraulics Institute. The channel received water from a tank placed on the piezometric tower; the tank had a filling channel which was quite long, to ensure a regular flow. The shedding of vortices was recorded by Dantec's LDA system (containing a coaxial lens and a Bragg's cell) consisting of a 5 mw He-Ne laser mounted on a manual traversing system. The photomultiplier, with forward scatter, was connected to the frequency-shifter and the tracker. The analogic signal from the tracker was sent to the acquisition card of a PC. The data were processed by software supplied by Dantec. While these experiments were being carried out, the vortex filaments were visualised by means of the usual hydrolysis technique (Buffoni 1995, 1996). During the second phase, using an appropriate device, vortex shedding was studied for Reynolds numbers below the critical value. At an earlier stage, it had been discovered that it was possible to achieve shedding under subcritical conditions, by making the cylinder vibrate transversally, with a low amplitude and a suitable frequency. As a result, a trolley on rails was planned and constructed with great care and precision; the cylinder, attached to the trolley, was made to vibrate at a predetermined amplitude and frequency, which was measured by a Ono-Sokki LD 1100S-005 (using infrared beams) vibrometer and a Tektronics 504A frequency meter. In this case too, through visualisation, it was possible to check the formation of vortex filaments, which disappeared as soon as the trolley came to a halt (Buffoni 1997).

During the third phase, the images of particles suspended in the fluid were recorded, so as to make available, for future reference, information on the real circulation generated. The experimental apparatus consisted of a trolley running on rails set about 1.50 m apart, while the electronic control unit was able to regulate the speed of the trolley. The cylinder, which was attached to it, moved under fluid in a pool of still water. The velocity field was illuminated by an intermittent light sheet obtained by a He-Ne laser and a cylindrical lens. Particles of TiO2 or, alternatively, glass microspheres, with a silver surface and a diameter of 10 μm, were added to the water. The images were recorded with a CCD SBIG ST5 camera and sent to a PC. Afterwards, measurement of the coordinates of points (measurements similar to those made in astrometry) could be performed.

Experimental Results

On the basis of over two hundred completed experiments, a notable dispersion was discerned, as is well known, in the first phase, if the data are displayed, diagrammatically showing the number of Strouhal against that of Reynolds. On the other hand, the results reveal a marked linearity if they are plotted with the shedding frequency as a function of the velocity of flow. As a result, the following linear law, which has never been presented in this form before, turns out to be in excellent agreement with the experimental data (Fig. 1), both for high and for low Reynolds numbers, with general validity (Buffoni 1995, 1996):

$$U - U_0 = 2\pi\alpha d(f - f_0) \tag{1}$$

where $\alpha = \pi/4$ indicates the angular amplitude (Buffoni 1999), f the shedding frequency, d the diameter of the cylinder, and f_0 and U_0 the critical values for frequency and velocity respectively, giving rise to the phenomenon. They are given by the following empirical relationship:

$$U_0 = 49\frac{\nu}{d} \tag{2}$$

$$f_0 = 0.12\frac{U_0}{d} \tag{3}$$

Thus, through equations (1),(2) and (3), we are able to forecast the frequency of vortex shedding for a generic cylinder in any flow condition. In addition, analysis of critical values gives the following equation:

$$U_0^2 = 406\nu f_0 \tag{4}$$

where ν is the kinematic viscosity of the fluid. Besides, Taylor's hypothesis on the dragging of turbulence at the mean flow velocity, allows us to derive from (4) the wavelength λ_0, in other words the space between two successive vortices:

$$\lambda_0 = 406\frac{\nu}{U_0}. \tag{5}$$

In the second phase of the experiments, under subcritical conditions, that is, when $U < U_0$, the phenomenon no longer occurs spontaneously, but it is still possible to achieve vortex shedding by making the cylinder vibrate at a frequency of f. The most intensive spectrum band appears when frequency, velocity and critical velocity are correlated according to the following equation:

$$U^2 = 406\nu f - L \tag{6}$$

where $L = (U - U_0)^2$. By now attentive readers will have noted a close analogy, even if a formal one, between (6) and the photoelectric effect. The recording of images with a CCD camera allows us to acquire an explanation of the fact that vortex filament are shed only with Reynolds numbers over 25. For values lower that these, the pair of stationary vortices located downstream, with respect to the cylinder, are small (Fig. 2). In the following section we will see how the previous empirical equations can be explained in terms of an effective, coherent quantum theory.

Basic Equations

In a Newtonian fluid, with a kinematic viscosity ν, the equations of Navier-Stokes hold; using the summing convention, they can be written in the following tensorial form ($i, j = 1 \div 3$):

$$\frac{1}{\varrho}\frac{\partial p}{\partial x_i} + u_j\frac{\partial u_i}{\partial u_j} + \frac{\partial u_i}{\partial t} = \nu\frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{7}$$

For null pressure gradients, the three previous equations, in very slow motion approximation, are reduced to the Fourier equation that gives solutions valid for phenomena that spread through space without any wave propagation. If, on the other hand, we consider a point outside the boundary layer, where the gradients of the transverse velocity are null, or within the viscous sub-layer where they are constant, the right-hand side in equations (7) is null, and the first one is reduced to Euler's equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} \tag{8}$$

which constitutes Taylor's hypothesis on turbulence dragging by mean flow. By deriving both sides of the equation, first with respect to x and then with respect to time, on the basis of the theorem on the reversibility of the order of derivation, we obtain D'Alembert's linear equation:

$$\frac{\partial^2 u}{\partial t^2} = U^2\frac{\partial^2 u}{\partial x^2} \tag{9}$$

where U indicates the phase velocity equal to mean flow velocity. In this way Taylor's hypothesis is justified, as the solutions of (9) centre on $(x - Ut)$, and indicate fluctuations that are dragged downstream by the flow itself. In general terms it may be postulated that (8) takes the form:

$$\frac{\partial u_i}{\partial t} = -u_i\frac{\partial u_i}{\partial x_i}, \tag{10}$$

which enables (9) to be formulated in general terms, on the hypothesis of the isotropy of the phase velocity:

$$\frac{\partial^2 u}{\partial t^2} = U^2 \bigtriangledown^2 u. \tag{11}$$

For a predetermined value of x, the solution of (9) is reduced to a sinusoid:

$$u(t) = a\mathrm{e}^{i\omega t}. \tag{12}$$

In addition, this accounts for the linearity of the experimental results and the introduction by Levi (1983) of the model of a classic oscillator. Equation (12) does, in fact, also express the solution for a harmonic oscillator which has amplitude a and angular frequency ω. For such an oscillator, the kinetic energy E and potential energy V, with reference to the unity of mass, can be expressed by:

$$E = \frac{1}{2}U^2 \tag{13}$$

$$V = \frac{1}{2}a^2\omega^2. \tag{14}$$

Moreover, mean energy, kinetic energy and potential energy are equal, so that total mean energy T can be expressed by:

$$T = U^2 = a^2\omega^2. \tag{15}$$

Thus, considering the proportionality coefficient α between the oscillation amplitude and the cylinder's diameter d, we obtain the result:

$$U = 2\pi\alpha d f. \tag{16}$$

If we now introduce the minimum values U_0 and f_0 above which vortex shedding occurs - values given by the empirical equations (2) and (3) - we then obtain the general equations (1) in line with the experimental data. The critical value U_0 and f_0 cannot be explained by the model of the classic oscillator; one must first introduce the quantisation hypothesis, that is, the energy of the oscillator is not continuous, but distributed at discrete levels.

Cell dimensions in the phase space

The phase space comprises six dimensions, three spatial coordinates x_i and three velocity components u_i. To carry out a statistical analysis on the system's state of movement, this space must be divided into small cells and the points that represent the system which fall within each cell must be counted. The size of each cell is arbitrary for a perfect fluid; it is determined on the basis of purely practical considerations. On the other hand, for a Newtonian fluid at a given temperature, and hypothetically, subject to turbulent movement, the volume of the cells must be ($i = 1 \div 3$):

$$\triangle x_i \triangle u_i \geq R\nu \tag{17}$$

which is the condition for there to be turbulence; when there is equality, the equation expresses transition. R stands for a critical Reynolds number which, when multiplied by the kinematic viscosity ν, gives the value of the constant. The volume of the cells cannot be arbitrary, because, if (17) is not satisfied, that is, if their volume turns out to be lower than the constant, one is considering zones of fluid in laminar motion. As can be seen from equation (5), experience suggests that transition occurs through oscillations of length λ correlated with velocity by the following equation (analogous with De Broglie's equation):

$$\lambda = \frac{\mathrm{k}\nu}{u} \tag{18}$$

where k represents a universal constant that is valid for all Newtonian fluids. Due to the fact that we are dealing with wave phenomena, we will consider Fourier's fundamental theorem, which states that between the length $\triangle x$ of a wave packet and the corresponding spectrum band $\triangle\kappa$, the following relationship holds:

$$\triangle x_i \triangle \kappa_i \geq \frac{1}{2} \tag{19}$$

where $\kappa = 2\pi/\lambda$ indicates the wave number.

By introducing the wave number in (18) and giving this value in (19) we obtain:

$$\triangle x_i \triangle u_i \geq \frac{\mathrm{k}\nu}{4\pi}. \tag{20}$$

The universal constant k, at the present state of knowledge, has a value of 2,550, as a result of which constant R in (17) has a value of 203. The constant that is characteristic of a given Newtonian fluid can be indicated by $k = \mathrm{k}\nu$ which has the dimensions of an action per unit of mass. If we indicate that $k_{\o} = k/2\pi$ (20) will take on the following form:

$$\triangle x_i \triangle u_i \geq \frac{k_{\o}}{2}, \tag{21}$$

so that the constant $\mathrm{k}/2\pi$ is equal to 406. The physical meaning of (20) and (21) is as follows: given a fluctuation velocity $\triangle u$, the energy supplied by mean movement is insufficient to obtain wave packets with dimensions below $\triangle x$; the viscosity of the fluid does not permit this.

The Wave Equation

Velocity u satisfies D'Alembert's equation (11), according to which the kinetic energy of the fluid is proportional to u^2 at every point. So, by analogy, we may posit a wave function ψ, which will generally be complex, called "probability amplitude", such that its module square $\psi\psi^*$ does not express energy, but a density of probability. The problem, therefore, is not that of knowing the real value of the energy, but the probability that, at certain point in the field of movement, there will be a spontaneous fluctuation, or burst. What is introduced here, then, is a probability conception of hydrodynamics. Under critical conditions the event will take place at some point in the movement field; as a result, the probability P over the whole domain Ω occupied by the fluid will be:

$$P = \int_{\Omega} \psi\psi^* d\Omega = 1 \tag{22}$$

which represents the normalisation condition. The wave function ψ itself satisfies D'Alembert's equation as expressed in the general form:

$$\bigtriangledown^2 \psi = \frac{1}{U^2} \frac{\partial^2 \psi}{\partial t^2}. \tag{23}$$

If we try to find a solution that is independent of time, a sinusoid can be introduced into (23); ψ_o will then only be a function of the spatial coordinates, so giving us the result:

$$\bigtriangledown^2 \psi_o + \kappa^2 \psi_o = 0. \tag{24}$$

Introducing De Broglie's equation $\lambda = k/u$ and supposing that the kinetic energy $U^2/2 = (E - V)$, we obtain:

$$\bigtriangledown^2 \psi_o + \frac{2}{k_{\o}^2}(E - V)\psi_o = 0. \tag{25}$$

Lastly, adding the Hamilton's operator:

$$H_{op} = -\frac{k_{\o}^2}{2} \bigtriangledown^2 + V \tag{26}$$

we obtain the definitive result:

$$H_{op}\psi_o = E\psi_o \tag{27}$$

that is, Schödinger's equation independent of time and valid only for monochromatic waves. To obtain this in a general case, we can utilise the expression for the wave function:

$$\psi = \psi_o \mathrm{e}^{-i\frac{E}{k_\varnothing}t}.$$

We can now derive this expression with respect to time:

$$\frac{\partial \psi}{\partial t} = -i\frac{E}{k_\varnothing}\psi.$$

In addition, ψ must satisfy the equation:

$$\bigtriangledown^2 \psi + \frac{2}{k_\varnothing^2}(E - V)\psi = 0.$$

Eliminating energy E from these two last equations, we obtain:

$$-\frac{k_\varnothing^2}{2} \bigtriangledown^2 \psi + V\psi = ik_\varnothing \frac{\partial \psi}{\partial t} \tag{28}$$

that is, Schödinger's temporal equation, which is, therefore, valid for any wave.

By now the reader will already have recognised that equations (21) and (24) are analogous with Heisenberg's inequality and with Schödinger's equation for quantum mechanics, respectively; the analogy is only formal, as it is clear that we are in the field of macroscopic phenomena independent of Planck's constant. However, for a Newtonian fluid at a certain temperature, there is a constant k dependent on viscosity, which governs the phenomenon.

Equation (24) has been successfully applied to the transition to turbulence in main inner motions (Buffoni, 1995); in the next section it will be used to explain the triggering mechanism for vortex shedding.

The Quantised Oscillator

Wave equation (25) is valid for a harmonic oscillator with an angular frequency ω, where the potential energy is:

$$V = \frac{1}{2}\omega^2 x^2$$

This is a Sturm-Liuville problem; once solved, it supplies the succession of eigenvalues or energy levels of the oscillator itself:

$$\epsilon_n = k_\o \omega (\frac{1}{2} + n) \tag{29}$$

where $n = 0, 1, 2, 3...$, and the corresponding eigenfunctions:

$$\psi_n(x) = \left(\frac{\omega}{\pi k_\o}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} \mathrm{e}^{-\frac{\omega}{2k_\o}x^2} H_n\left(x\sqrt{\frac{\omega}{k_\o}}\right) \tag{30}$$

where H_n, are the Hermite polynomials. Furthermore the probability density is given by:

$$p_n(x) = \psi_n^2.$$

The first case, with $n = 0$, is shows in Fig. 3; in comparing this with Fig. 2a, it may be noted that where the probability density is not equal to zero, a couple of stationary vortices really exist. The eigenvalues are found at a series of intervals - each successive interval an incremental multiple of the first - from the quantity $k_\o \omega$, which does, in fact, represent the energy quantum. In addition, for (29) the energy levels of the oscillator become more and more closely clustered as k becomes smaller, and as, therefore, ν too becomes smaller, until continual energy returns in the case of a perfect fluid. When the system turns out to be unstable at a given level, it falls back to an immediately lower one, giving off a quantum of energy, $k_\o \omega$; the phenomenon of vortex shedding is therefore due to the transition between two consecutive energy levels. In general, the most probable excited level is the first, that is, with n=1; as a result, transition mainly occurs between this and the fundamental level. If we remove from (29) the energy of the fundamental state, we obtain the series:

$$\epsilon_n = k_\o \omega n \tag{31}$$

where $n = 1, 2, 3...$ On the hypothesis that a vortex shedding takes place with frequency f equal to the angular frequency ω:

$$\epsilon_n = k_\o f n. \tag{32}$$

In the fundamental state ($n = 1$) we therefore obtain:

$$\epsilon = k_\o f \tag{33}$$

where $\epsilon = U^2$. Equation (33) represents the energy quantum, that is, the minimum amount of energy that must be supplied to the system for the phenomenon begin. The constant $k = 406\nu$ is in full agreement with the experimental results expressed by equation (4), as can be noted in figure 4.

The oscillator can enter into various different states, each of which has an energy equivalent. It is, therefore, necessary to evaluate the mean energy possessed by the oscillator when it is in the most probable state. We will not present the complete description given by Fermi (1934). We will only give the results: the mean total energy ϵ for a quantised oscillator in contact with a source of energy with a value of T is given by the following equation:

$$\epsilon = \frac{k_\o f}{\mathrm{e}^{\frac{k_\o f}{T}} - \mathit{1}}. \tag{34}$$

If we allow the energy quantum to tend to zero, it may be observed that ϵ tends to T. The mean energy approximates to the case of the classic oscillator which, when in statistical equilibrium, has the same energy as the source with which it is in contact. In concrete terms, for mean energy values greater than the quantum $k_\o f$, there is a substantially classic form of behaviour, whereas for lower values it deviates from the classic law according to equation (34). In the last analysis, vortex shedding begins, and the critical state is, therefore, reached, when the system possesses an energy quantum, that is, $k_\o f/T = 1$. We may therefore use equations (34) and (15) in theoretically calculating the critical values for frequency and velocity. In fact, taking into account (34) and (15) ($\epsilon = T = a^2\omega^2$), for $k_\o f/T = 1$ and $a = d$, we obtain the critical frequency f_o:

$$f_o = \frac{k_\o}{(2\pi d)^2(\mathrm{e} - 1)}. \tag{35}$$

In addition, as $f_o = U_o^2/k_\o$, we can now calculate the critical velocity U_o:

$$U_o = \frac{k_\o}{(2\pi d)\sqrt{\mathrm{e} - 1}}. \tag{36}$$

The values calculated using equations (35) and (36) are in accordance with the experimental data (Buffoni 1996). We can, however, make them dimensionless by calculating the critical numbers of Reynolds and Strouhal:

$$Re_o = \frac{\mathrm{k}/2\pi}{2\pi\sqrt{\mathrm{e}-1}} = 49 \tag{37}$$

$$St_o = \frac{1}{2\pi\sqrt{\mathrm{e}-1}} = 0.12, \tag{38}$$

in full accordance with all known experiments.

In subcritical conditions, that is, for velocity $U < U_o$, the phenomenon does not occur spontaneously, but it is possible to achieve vortex shedding by making the cylinder vibrate at an appropriate frequency. The previously stated equation (33) must therefore be modified to take account of the work L needed to reach the critical state:

$$\epsilon = k_{\o} f - L \tag{39}$$

where f indicates both the shedding frequency and the frequency at which the cylinder is made to vibrate. In other words, to obtain vortices dragged by the flow and, therefore, with total mean energy $\epsilon = U^2$, what must be supplied is a quantum of energy diminished by the work L needed to extract them. The work of extraction or, more simply, the work required to reach the critical state, is:

$$L = (U_o - U)^2. \tag{40}$$

In this case too the constant $k_{\o} = 406\nu$ is in excellent agreement with the experimental data, as can be seen from the empirical equation (6) and from figure 5.

The Oscillations Spectrum in the Boundary Layer

In the viscous boundary layer, there may be oscillations with a continuous spectrum. We may think of innumerable oscillators with total energy distributed among them. Therefore, to calculate the number of these oscillators, which have a frequency lying between f and $f + df$, we will utilise phase space, especially space of the momenta q. The number of systems, in this case oscillators possessing momenta q, will be equal to the volume differentiating the two spheres with radius q and $q + dq$, that is, $4\pi q^2 dq$. It is already known that the calculation gives the number of oscillators whose frequency falls between f and $f + df$:

$$dN = \frac{8\pi}{\upsilon^{*3}} f^2 df. \tag{41}$$

Thus dN represents the number of oscillators belonging to the frequency interval df, so that if their mean energy is $T = \upsilon^{*2}$, where υ^* stands for shear velocity, it will be true that the energy contained in that interval will be $d\epsilon = TdN$, that is:

$$d\epsilon = \frac{8\pi}{\upsilon^{*3}} T f^2 df \tag{42}$$

but as $d\epsilon = u(f,T)df$, the energy spectrum included within the boundary layer is:

$$u(f,T) = \frac{8\pi}{\upsilon^{*3}} T f^2. \tag{43}$$

This equation is analogous with that obtained by Rayleigh and Jeans for classic electromagnetic radiation. The spectrum defined by (43) may be in agreement with the experimental data only for low frequency levels; (43) is, in fact, a rising monotone function, and its integral, that is, total energy over all frequencies, is infinite, which is clearly absurd. The situation is therefore similar to that encountered in the beginning of the century 1900s when Planck, to solve the problem of radiation from a black body, introduced the hypothesis of quanta. In this case too one must use a quantum oscillator, the only difference being that, as this is a macroscopic phenomenon, it does not depend on Planck's constant, but on the universal constant that is valid for all Newtonian fluids: $k = 2,550\nu$. Using the quantum hypothesis, the count of the number of oscillators falling in the interval between f and $f + df$ does not change; all that changes is the mean energy, which must now be expressed by (34), so that energy per volume unit within the boundary layer is:

$$u(f,T) = \frac{8\pi}{\upsilon^{*3}} \frac{kf^3}{\mathrm{e}^{\frac{kf}{T}} - 1}. \tag{44}$$

This differs from Planck's formula only in terms of the constant k. As may be seen from figure 6, (44) has a maximum value, and the area subtended by the curve, that is, total energy for all frequencies, has a finite value. The maximum frequency is given by the equation:

$$f_{max} = \alpha \frac{T}{k} \tag{45}$$

where α is the root of the equation:

$$(3-\alpha)\mathrm{e}^{\alpha}=3,$$

whose solution is $\alpha = 2.821$, so that from (45) we now obtain:

$$\lambda^{+}=\frac{\lambda_{max}\upsilon^{*}}{\nu}=904. \tag{46}$$

It may be noted (45) is analogous with the Wien shift law. Many authors note that in the boundary layer there are helicoidal structures called "rolls" whose dimensionless length is $800 \leq \lambda_x^+ \leq 1,000$, which is, in fact, compatible with equation (46). In the boundary layer, contractions within these rolls produce velocity fluctuations corresponding to the maximum value of the spectrum given by the quantum theory. The wave length of the rolls, in a direction transversal to the motion λ_z, has a value between $100 \leq \lambda_z \leq 300$, and is therefore in accordance with equation (17).

Conclusions

While we have been waiting for supercomputers capable of solving the Navier-Stokes equations, a large-scale experimental research project has been carried out at this Hydraulics Laboratory, with the aim of understanding the simplest source of vorticity in nature - a cylinder immersed in a uniform flow -. While this research was under way, four discoveries, each so far unreported and unanticipated, have been made. The first innovation consists of the general law (1), which is valid for any Reynolds number and can be derived from the model for the harmonic oscillator.

The second comprises the application of the theory of quanta to the oscillator; surprisingly, it is able to account for the critical values for the onset of the phenomenon. The energy of the oscillator cannot be continuous; it has discrete values, so that the quantum hydrodynamics must be considered. The third, which has a number of possible practical applications, refers to the possibility, for Reynolds numbers below the critical one, of obtaining vortex shedding by means of an appropriate transversal vibration of the cylinder.

The fourth innovation is the discovery of equation (39), which expresses the relationship between the energy of the flow affecting the oscillator and the energy quantum as reduced by the work needed to return the system to a critical condition. In other words, the mean value of the energy possessed by the vortices is equal to the energy quantum, less the work needed to extract them.

The results of these discoveries is that of making intelligible the mechanisms that initiate the turbulence generated by a single source. This model can successfully be applied to many sources, and, therefore, to many oscillators that have a continuous spectrum, instead of a band spectrum. As seen above, a phenomenon of this type is found in the viscous boundary layer.

Thus the quantum Hydrodynamics of Newtonian fluids has a solid foundation in nature, and may lead to important developments.

-+ Bibliography

Buffoni E. 1993. Distacco di vortici a bassi numeri di Reynolds. *AIVELA Meeting Capri.*

Buffoni E. 1995a La transizione alla turbolenza. *La ricerca ed i ricercatori della Facoltá di Ingegneria.* T.E.P. Pisa 19-31.

Buffoni E. 1995b. La legge generale sul distacco dei vortici da corpi cilindrici. T.E.P. Pisa.

Buffoni E. 1996. Ricerca sperimentale sul distacco dei vortici. T.E.P. Pisa.

Buffoni E. 1997. Il distacco dei vortici in condizioni subcritiche. T.E.P. Pisa.

Buffoni E. 1998. I nuovi aspetti dell'emissione di vorticitá. *Acta XXVI Hydraulics and Hydraulic constructions Meeting, Catania, IV* 243-254.

Buffoni E. 1999. La natura della turbolenza. Edizioni E.T.S. Pisa pp. 159.

Fermi E. 1934. Molecole e cristalli - parte IIIa: La statistica della teoria dei quanti. Zanichelli, Bologna.

Friehe C.A. 1980. Vortex shedding from cylinders at low Reynolds numbers.*J.F.M.* **100** 237-241.

Levi E. 1983. A universal Strouhal law. *J.Eng. Mech. A.S.C.E.* **109** 718-727.

Levi E. 1983. Oscillatory model for wall-bonnded turbulence *J. Eng. Mech. A.S.C.E.* **109** 728-740.

Roshko A. 1954. On the development of turbulent wakes from vortex streets *N.A.C.A.* **1191** 1-24.

Son D.T. 1999. Turbulent decay of a passive scalar in the Batchelor limit: Exact result from a quantum-mechanical approach. *Physical Review E* **59** R3811-R3814.

Tritton D.J. 1971. A note on vortex streets behind circular cylinders at low Reynolds numbers. *J.F.M.***45** 203-208.

Van Atta C.W., Gharib M. 1987. Ordered and chaotic vortex streets behind circular cylinders at low Reynolds numbers. *J.F.M.* **174** 113-133.

Williamson C.H.K. 1996 Vortex dinamics in the cylinder wake. *Ann. Rev. Fluid Mech.* **28** 477-539. -+