Analytical evaluation of the stress fields in tapered box girders

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Tapered beams are widely used in both civil and industrial engineering for a more efficient exploitation of material in comparison to prismatic beams. In the wind energy sector, tapered box girders are commonly used as the main structural elements of composite wind turbine blades [1].

In the scientific literature, it is long known that the internal distribution of stresses in tapered beams may differ significantly from that present in prismatic beams [2]. In 1932, Bleich presented an extension of Jourawski's formula showing that in beams of variable cross section, shear stresses arise as a result of not only shear force, but also bending moment and axial force [3].

Notwithstanding the above, standard design procedures do not take into account the effects of taper on the stress distribution in a beam, with consequent possible under- or over-estimation of the stress states acting in a real structure [4]. Only recently, researchers have focused their attention on the effects of taper in beams of variable cross section with a view towards the practical design of structures. A broad review of the literature on non-prismatic beams can be found in the recent work by Balduzzi et al. [5].

In this work, we further extend Bleich's shear stress formula to tapered beams subjected to distributed loads. Furthermore, we specialise the obtained result to the analysis of tapered box girders. By introducing suitable assumptions on the stress field in a thin-walled cross section, from the integration of Cauchy's equilibrium equation, we determine analytical, yet approximate, expressions for all the stress components. Thus, the contributions stemming from the effects of taper and the differences with the stress fields in prismatic beams are clearly highlighted. Comparison of the analytical predictions with the numerical results of finite element analyses obtained through the commercial software Abaqus shows an excellent agreement, thus validating the proposed approach [6].

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