Università di Pisa

Esame di SCIENZA DELLE COSTRUZIONI *– Parte I*
Corso di Laurea in Ingegneria Aerospaziale

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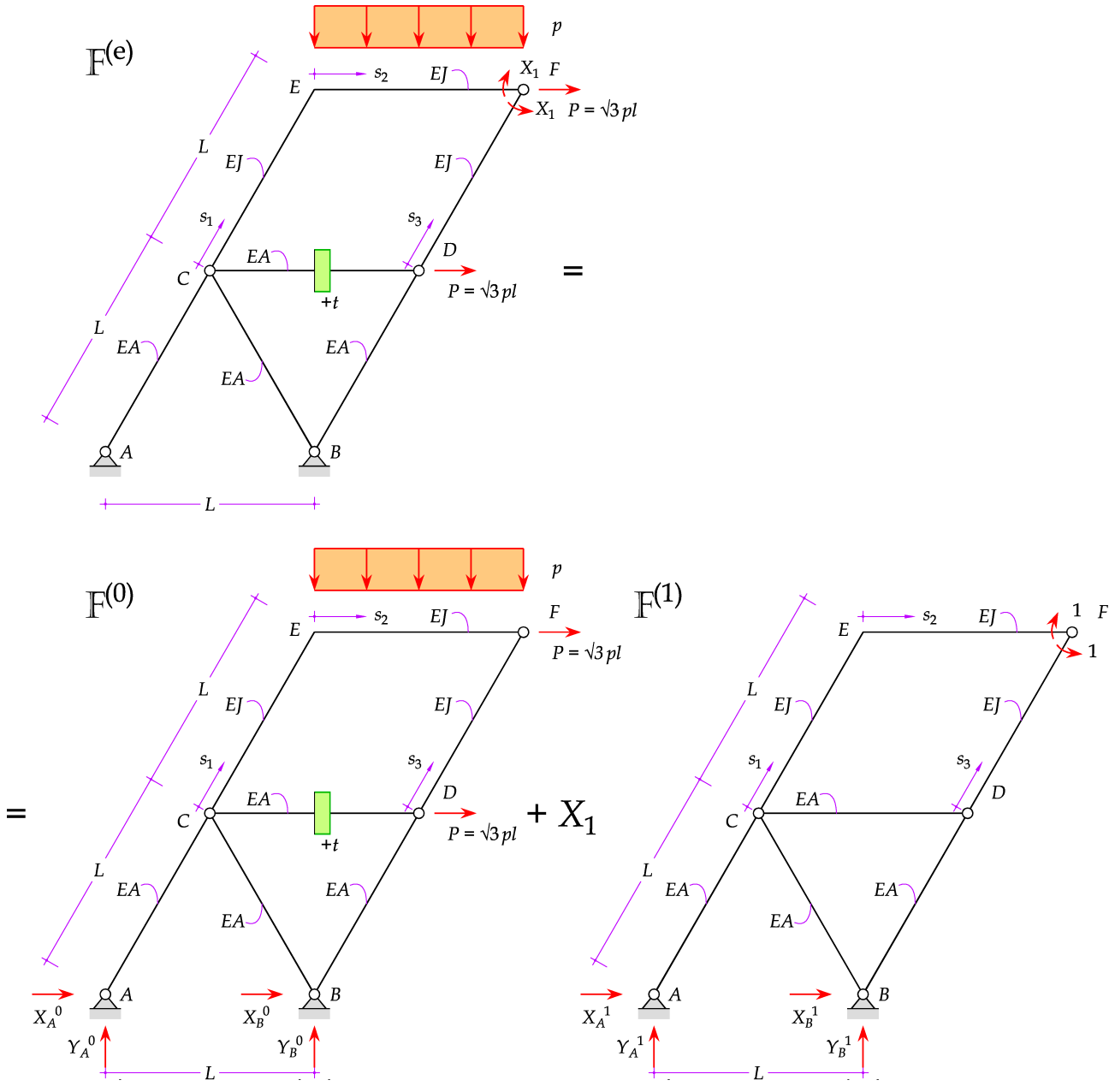
Prova scritta del 22 luglio 2017 – Soluzione

1) Metodo delle forze



Reazioni vincolari:

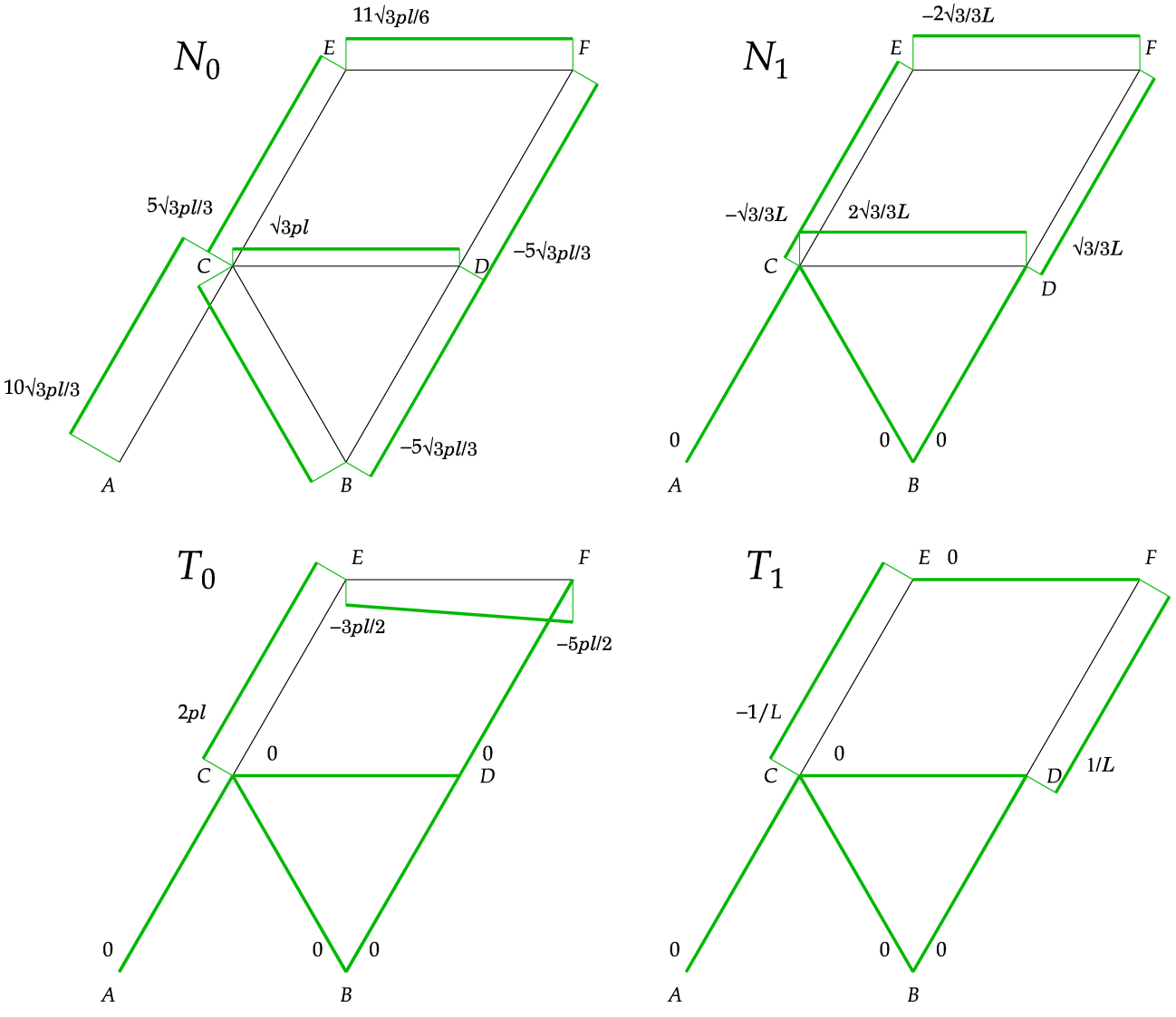
$$X_A^0 = -\frac{5\sqrt{3}}{3}pL, \quad Y_A^0 = -5pL, \quad X_B^0 = -\frac{\sqrt{3}}{3}pL, \quad Y_B^0 = 6pL;$$

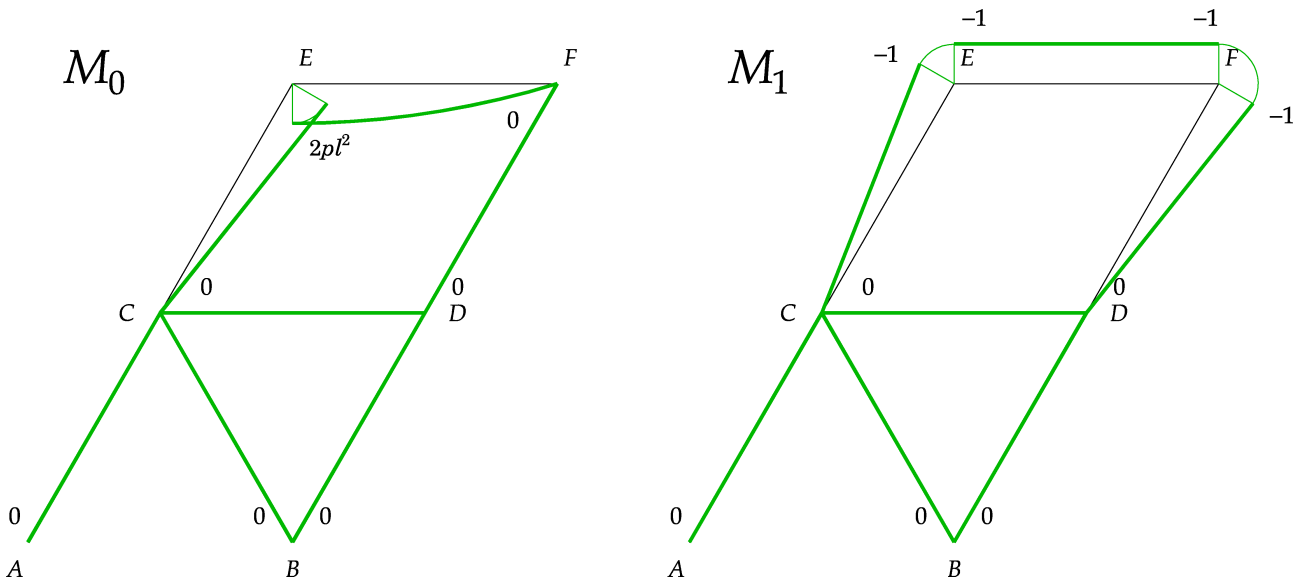
$$X_A^1 = Y_A^1 = X_B^1 = Y_B^1 = 0.$$

Caratteristiche della sollecitazione:

Tratto	N_0	T_0	M_0	N_1	T_1	M_1
AC	$\frac{10\sqrt{3}}{3}pL$	0	0	0	0	0
BC	$-\frac{7\sqrt{3}}{3}pL$	0	0	0	0	0
BD	$-\frac{5\sqrt{3}}{3}pL$	0	0	0	0	0
CD	$\sqrt{3}pL$	0	0	$\frac{2\sqrt{3}}{3L}$	0	0
CE	$\frac{5\sqrt{3}}{3}pL$	$2pL$	$2pLs_1$	$-\frac{\sqrt{3}}{3L}$	$-\frac{1}{L}$	$-\frac{s_1}{L}$
DF	$-\frac{5\sqrt{3}}{3}pL$	0	0	$\frac{\sqrt{3}}{3L}$	$\frac{1}{L}$	$\frac{s_3}{L}$
EF	$\frac{11\sqrt{3}}{6}pL$	$-\frac{3}{2}pL - ps_2$	$-\frac{3}{2}pLs_2 - \frac{1}{2}ps_2^2 + 2pL^2$	$-\frac{2\sqrt{3}}{3L}$	0	−1

Diagrammi:





Coefficienti di Müller-Breslau:

$\eta_1 = 0$

$$\eta_{10} = \int_0^L M_{CE}^1(s_1)\frac{M_{CE}^0(s_1)}{EJ}ds_1 + \int_0^L M_{EF}^1(s_2)\frac{M_{CE}^0(s_2)}{EJ}ds_2 \; + N_{CD}^1\varepsilon_{CD}^0 = -\frac{7}{4}\frac{pL^3}{EJ} + \frac{2\sqrt{3}}{3}\alpha t$$

$$\eta_{11} = \int_0^L \frac{[M_{CE}^1(s_1)]^2}{EJ}ds_1 + \int_0^L \frac{[M_{EF}^1(s_2)]^2}{EJ}ds_2 \; + \int_0^L \frac{[M_{DF}^1(s_3)]^2}{EJ}ds_3 \; = \frac{5}{3}\frac{L}{EJ}$$

Incognita iperstatica:

$$\eta_1 = \eta_{10} + X_1\eta_{11} = 0 \;\Rightarrow\; X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{\frac{7}{4}\frac{pL^3}{EJ} - \frac{2\sqrt{3}}{3}\alpha t}{\frac{5}{3}\frac{L}{EJ}}$$

2) Metodo della linea elastica

Equazioni differenziali:

$$EJv_1^{IV}(s_1) = 0, \;\; 0 < s_1 < L, \qquad EJv_2^{IV}(s_2) = p, \;\; 0 < s_2 < L, \qquad EJv_3^{IV}(s_3) = 0, \;\; 0 < s_3 < L.$$

Condizioni al bordo:

$$v_1(0) = 0, \quad v_1^{II}(0) = 0, \;\; v_1^I(L) = v_2^I(0), \;\; v_1^{II}(L) = v_2^{II}(0),$$

$$v_2(0) = \frac{1}{2}v_1(L), \quad v_2(L) = \frac{1}{2}v_3(L), \quad v_1(L) = v_3(L),$$

$$v_3(0) = \frac{2\sqrt{3}}{3}\alpha t L, \quad v_3^{II}(0) = 0, \;\; v_3^I(L) = v_2^I(L), \;\; v_3^{II}(L) + v_3^{II}(L) = 0,$$

$$EJv_1^{III}(L) + EJv_3^{III}(L) + \frac{\sqrt{3}}{2}P + \frac{1}{2}pL = 0\,.$$