ABSTRACT. Plunge pools are currently designed with a limited number of formulae, most of which are either outdated or dimensionally incorrect. Based on a large data accumulation collected at VAW in 2003, a selection of formulae was tested to obtain indications on their performance. It was observed that some formulae may be recommended, but most must be limited for application. The present paper may shed new light on the research directions required for future projects, and the limitations of the available methods for predicting the maximum plunge pool depth. It was also observed that a number of parameters considered important during the VAW research was so far overlooked, such as the relative tailwater depth or the effect of impact jet shape. The results of the present study may guide designers in the selection of their model equations.

1 INTRODUCTION

Plunge pool scour was considered in the past extensively by adopting the Froude similarity law (Hager 2000). Recently, the present authors have conducted a large experimental research project involving some 435 experiments to assess the effects of the following main parameters: (1) Approach velocity, (2) Sediment size, (3) Sediment gradation, (4) Jet impact direction relative to the horizontal, (5) Jet air content, and (6) Tailwater elevation above sediment bed (Pagliara et al. 2004, 2005).

The hydraulic literature contains five main experimental studies in which plunge pool scour was investigated and where formulae mainly for the maximum scour depth were developed: (1) Mason and Arumugam (1985), and Mason (1989); (2) D’Agostino (1994) and D’Agostino and Ferro (2004); (3) Schoklitsch (1932) and Veronese (1937); (4) Martins (1975); and (5) Kotoulas. The following intends to compare their formula with data sets elaborated at Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie VAW, ETH Zurich, Switzerland. First, the authors’ experimental setup is described, then the previous formulae are presented and compared with the authors’ data previously mentioned.

2 AUTHORS’ EXPERIMENTAL SETUP

Figure 1 shows a definition sketch in which the main parameters are explained. The VAW setup involved a circular pipe where an air-water high-speed jet was generated to circumvent problems with a ski jump at the end of a spillway. Accordingly, an almost homogeneous air-water jet was issued directly onto a sediment bed instead of a spillway flow deflected by a ski jump that travels through the atmosphere to eventually impact a rocky surface. The VAW setup may thus be considered a simplification of a real model; it allowed the exact definition of the mixture impact velocity, the impact angle and the granulometric characteristics of a completely disintegrated rock surface. Based on preliminary experimentation, a number of features were investigated, relating mainly to scale effects due to the complicated three-phase flow structure. The VAW data set may thus be considered an alternative to the many case studies conducted in the past for a specific plunge pool, in which only a limited number of parameters had been analyzed.

Figure 1 involves the following main parameters: Water (subscript W) discharge $Q_W$, Air (subscript A) supply discharge $Q_A$, jet impact angle $\alpha$, tailwater depth $h_o$, maximum dynamic scour depth $z_m$, with $z_m$.
as the maximum static scour depth and maximum deposition height $z_m$, and possible water levels that

Figure 1 Definition sketch of plane plunge pool subjected with an air-water jet

produce submerged and unsubmerged approach flows, respectively. The densimetric particle Froude number $F_d = V/(g'd_{50})^{1/2}$ was identified as the main variable of the physical process, where $V$ is the jet impact velocity, $g' = [(\rho_s - \rho)/\rho]g$ the reduced gravitational acceleration with $\rho_s$ and $\rho$ as the sediment and fluid densities, respectively, and $g$ as gravitational acceleration. The relative scour depth $Z_m = z_m/D$ with $D$ as the pipe diameter may be expressed as (Pagliara et al. 2005)

$$Z_m = -0.34 \sin(\alpha + 22.5\degree)(1 + \beta)^{(1/0.30)}(0.12 \ln(1/T) + C_r)$$

(1)

where $\beta = Q_A/Q_w$ is the jet air content and $C_r = 0.45$ when the ridge remains unremoved, as is typical in applications. The relative tailwater depth $T = h_o/D$ has a significant effect on the maximum scour depth, whereas the effect of sediment granulometry remains insignificant provided the densimetric particle Froude number is employed based on the mean sediment size $d_{50}$. Note that no upstream discharge $Q_m$ was considered in the present research and that scour profiles of both the scour hole and the deposition ridge appear to be unavailable. Pagliara et al. (2004, 2005) furnish both the effect of $Q_m$ and ridge removal on the maximum scour depth, and the generalized profile of the scour hole $z(x)$. It should be noted that the description of the ridge profile is complicated by the combined effect of scour hole advance and ridge erosion by the tailwater elevation. No final proposal for the ridge profile is so far available, therefore, because additional research appears to be necessary. Pagliara et al. (2004) furnish preliminary information on the temporal advance of the maximum scour depth based on a time scale within which end scour occurs.

3 EXISTING SCOUR FORMULAE

Mason and Arumugam (1985), Mason (1989)

This study involves 26 prototype data and 47 model data for free overfalls, low level outlets, spillway flip buckets, and tunnel outlets. With $q$ as specific water discharge, $H$ fall height of jet, $h_o$ tailwater depth and $z_s$ as the static (subscript $s$) scour depth after jet flow is stopped, the ranges of data include $0.325 \text{ m} < H < 2.15 \text{ m}$; $0.015 \text{ m}^2/\text{s} < q < 0.42 \text{ m}^2/\text{s}$; $0.001 \text{ m} < d_{50} < 0.028 \text{ m}$; and $0.071 \text{ m} < (z_s + h_o) < 1.175 \text{ m}$ for model tests, and $15.82 \text{ m} < H < 109 \text{ m}$; $2.36 \text{ m}^2/\text{s} < q < 220 \text{ m}^2/\text{s}$; and $6.70 \text{ m} < (z_s + h_o) < 90 \text{ m}$ for prototype data. The impact angles ranged from $25\degree$ to $85\degree$, and from $20\degree$ to $72\degree$ for model and prototype data, respectively. Mason and Arumugam (1985) proposed for the maximum static scour depth

$$z + h_o = K[\frac{g^x H^y h^{0.15}}{(g^{0.3} d_{50}^{0.1})}]$$

(2)

where $K = (6.42 - 3.10H^{0.10})$, $x = 0.60 - (H/300)$, and $y = 0.15 + (H/200)$ involve (dimensionally incorrect) the fall height $H$ [m].

Mason (1989) investigated the effect of air entrainment on plunge pool scour for a certain water discharge and a jet impact angle of $45\degree$. Two additional equations were presented to (2)
that account for the air discharge ratio, namely

\[ z + h_o = 3.39[1 + 0.30 q^{2/3} d_{50}^{0.06}] / (q^2 d_{50}^{0.06}) \quad (3) \]

\[ z + h_o = 3.27[H^{0.05} q^{0.60} h_o^{0.15}] / (q^2 d_{50}^{0.10}) \quad (4) \]

where the jet air content was determined with an expression by Ervine. The data of Pagliara et al. (2005) were processed with (4) under the assumption \( q=Q/D, \) \( H=(V_w \sin \alpha)^2/2g. \) Figure 2 shows the comparison between the (a) black water (BW) and (b) the white water (WW) data with (4). The BW plot demonstrates that the predicted data lie significantly above the measured data. It should be noted that Mason’s predictions do not include an effect of \( \alpha. \) The effect of air discharge seems to have no effect on the performance of (3). If static instead of dynamic scour data according to Pagliara et al. (2004, 2005) are considered, the agreement is slightly worse, yet essentially containing the same discrepancies. Accordingly, Equations (3) and (4) as also (2) based on a separate analysis may not be used to predict the VAW data set. The previous analysis was based on the assumption according to which the unit discharge is \( q=Q/D. \) The following refers to the alternative assumption \( q=Q/L \) where \( L=0.30 \) m was the jet width used by Mason (1989). Figure 3 indicates that the comparison between data and prediction improves, resulting in higher correlation coefficients, although many tests result in negative predicted scour depths.

D’Agostino (1994)

D’Agostino used a channel 0.50 m wide as in the VAW study in which a contracted, sharp-crested weir of relative widths \( K_o=0.41 \) and 0.71 times the channel width was placed. Two sediment mixtures were tested, namely 1 with \( d_{50}=4.1 \) mm, \( d_{90}=7.0 \) mm, and 2 with \( d_{50}=11.5 \) mm, \( d_{90}=17.6 \) mm. The unit discharges \( q \) ranged from 0.0167 \( m^3/s \) to 0.167 \( m^3/s \) producing static scour depths \( z_s \) between 0.045 m and 0.285 m. D’Agostino (1994) proposed the dimensionally incorrect formula

\[ z_s = (0.70 K + 0.58)(0.94q^{2/3} - 1.60d_{90}) \quad (5) \]

The VAW data set was applied to (5) with the same assumptions as for the Mason formulae plus \( b=D, \) in which \( b \) is the contracted jet width. Figure 4 shows a better agreement between data and prediction than previously for the Mason formulae, although predictions are generally too low. For the black water data, Fig. 4 (a) shows reasonable agreement, as do also the white water data in Figure 4 (b). When using \( q=Q/B \) with \( B \) as the channel width, the predictions tend to be generally too high.

D’Agostino and Ferro (2004) generalized their formula by including additional data from Veronese (1937), Bormann and Julien (1991), and Mossa (1998). Their revised formula reads

\[ z_{aw} = 0.54(b/w)^{0.597}(h_j/H)^{0.126} A_{50}^{0.544}(d_{90}/d_{50})^{0.856}(b/B)^{0.751} \quad (6) \]

where \( A_{50}=(q/w) \times 1/[g' d_{50}^{1/2}] \) is a modified densimetric particle Froude number and \( w=h_j+H \) is the total fall height. Figures 5 (a) and (b) for black and white water flows indicate a reasonable performance of (6) with the VAW data set provided \( q=Q/L \) is adopted, particularly for the weir width \( L=0.15 \) m. When using \( D \) instead of \( b \) the performance of (6) becomes much poorer, as previously noted already.

Schoklitsch (1932) and Veronese (1937)

Both of these formulae are considered actually historical, yet they were tested in the present analysis. Both authors used a similar experimental setup comparable to that of D’Agostino (1994), but had a full width weir. Schoklitsch (1932) used eight almost uniform sediments with \( 0.50 \) mm<\( d_{50}<16 \) mm, whereas those of Veronese (1937) varied within \( 9.1 \) mm<\( d_{50}<26.2 \) mm. The hydraulic heads varied, respectively, as 0.05 m<\( H<0.50 \) m and 0.65 m<\( H<1.32 \) m, and unit discharges were 0.003 \( m^3/s<q<0.10 \) \( m^3/s \), and 0.001 \( m^3/s<q<0.083 \) \( m^3/s \) for Schoklitsch and Veronese. The formulae proposed for maximum static scour depth were, respectively

\[ z_m + h_o = 4.75H^{0.20} q^{0.57} / d_{50}^{0.32} \quad (7) \]

\[ z + h_o = 3.62H^{0.225} q^{0.54} / d_{50}^{0.42} \quad (8a) \]

\[ z + h_o = 1.90H^{0.225} q^{0.54} \quad (8b) \]

where the Veronese equations (8a) and (8b) relate to model and prototype arrangements, respectively. Figure 6 (a) indicates poor
agreement of the white and the black water data with Schoklitsch’s prediction according to (7).

Figure 2  Comparison of VAW data sets with predictions of Mason (1989) for (a) blackwater $r^2 = 0.2$ and (b) whitewater data, $r^2 = 0.67$.

Figure 3  Comparison of VAW data with predictions of Mason (1989) for (a) BW data with $r^2 = 0.71$ and (b) WW data, $r^2 = 0.69$, for $q = Q/L$.

Figure 4.  Comparison of VAW data sets for static scour depths with predictions of D’Agostino (1994) for (a) blackwater $r^2 = 0.06$ and (b) whitewater data, $r^2 = 0.04$. 

Figure 5. Comparison of VAW data sets with predictions of D’Agostino and Ferro (2004) for (a) blackwater $r^2 = 0.10$ and (b) whitewater data, $r^2 = 0.06$ ($q = Q/L$, with $L = 0.15$ m).

Figure 6 Comparison of VAW data sets (dynamic scour) with predictions of Schoklitsch (1932) for (a) blackwater $r^2 = 0.42$, and (b) whitewater $r^2 = 0.40$ conditions.

Figure 7 Comparison of VAW data sets with predictions of Veronese (1937) for (a) blackwater $r^2 = 0.66$, and (b) whitewater $r^2 = 0.35$ conditions.
However, the Veronese prediction (7a) performs reasonably well with the VAW data, whereas (7b) for the white water data performs poor. Note that neither Schoklitsch nor Veronese accounted for an effect of air.

**Martins (1975)**

Martins developed a scour depth formula following indications of Indian and Russian researches. His dimensionally incorrect formula may be simply written as

$$z_s + h_0 = 1.50q^{0.60}H^{0.10}$$

(9)

Figures 8 (a) and (b) indicate that the black water data perform well with (9), whereas the performance of the white water data is somewhat poorer. Note also that three 30° impact angle data are located much too high for unknown reasons in the blackwater data comparison. Overall, the Martins formula predicts the VAW data reasonably well, given the simple relation among four parameters.

**Kotoulas (1967)**

Kotoulas (1967) proposed the following dimensionally incorrect formula for maximum depth of scour

$$z_s + h_0 = 0.78q^{0.70}H^{0.35}/d_{90}^{0.40}$$

(10)

Equation (10) is an empirical formula of general applicability; it was developed for a free overfall jet impacting an incohesive bed. Figure 10 compares his formula with our prediction as for the previously considered equations. It is seen that the whitewater data agree relatively well with his prediction, whereas there are larger deviations for the blackwater tests. Note also the similarity among (2), (7), (8), (9) and (10), in which the total depth ($z_s + h_0$) is expressed in terms of discharge $q$, head $H$ and sediment diameter $d_{50}$ or $d_{90}$. Again, no tailwater effect is included in (10).

**Figure 8** Comparison of VAW data sets with predictions of Martins (1975) for (a) blackwater $r^2=0.56$, and (b) whitewater $r^2=0.46$ conditions

**Figure 9** Experimental test with low tailwater
Plunge pool scour formulae

4 DISCUSSION OF RESULTS

The VAW research involves a total of more than 400 separate tests in which the governing parameters on maximum scour depth in a plunge pool arrangement were systematically investigated for an incoherent tailwater bed made up by sediment. It was interesting to note that most formulae so far presented involve a limited number of parameters, such as the total water depth above the scour hole \( z_s + h_o \) as originally introduced by Schoklitsch (1932), the unit discharge \( q \), the total head \( H \) and some determining sediment diameter \( d \).

The effect of impact angle is missing in all equations tested, and its effect is relatively small according to (1), provided \( 30^\circ \leq \alpha \leq 90^\circ \).

The effect of jet air content was accounted for only by Mason (1989), yet with a formula dimensionally incorrect. It was noteworthy that none of the formulae considered here involved an effect of tailwater level \( T \), as presented in (1). It appears obvious that scour depth reduces as the tailwater depth increases because of the ‘water cushion’, resulting in jet diffusion and a reduced impact onto the sediment bed. Most of the equations involve correctly \( d_{50} \) to account for the sediment size.

5 CONCLUSIONS

Plunge pool scour is a definite drawback of free jet trajectory spillways because a scour hole deeper than predicted may have an adverse effect on foundations. However, compared to standard energy dissipators such as stilling basins, plunge pools have significant advantages that may be of concern in the final selection of the type of structure for a particular site.

This research intended to compare some of the known formulae for the maximum plunge pool scour depth with data recently collected in a laboratory arrangement. It was found that the VAW data may only partially be predicted with these formulas, mainly because several of the formulae considered are dimensionally incorrect. Another main statement refers to the poor definition of the application ranges of these formulae. It became evident that some of the formulae are able to describe the data, yet their application range was unlimited. It was interesting to note that the Mason (1989) formula appears to compare best with the VAW data set if the effect of air entrainment is included, whereas the formula of D’Agostino (1994) produced the largest deviations. Given the simplicity of the Martins formula, the VAW data set is
reasonably well predicted. Predicting a scour hole thus needs a careful selection of the formulae. The present research would like to contribute to the safe design of hydraulic structures involving a plunge pool subjected with an air-water high-speed jet that is issued onto a completely disintegrated rock surface.

REFERENCES


Schweizerische Anstalt für das Forstliche Versuchswesen, 43(1). Beer: Zürich (in German)


