Problema 1 P.S S4C(II) - A - del 23/7/2010

3)
$$\sigma_{x,x} + \tau_{xy,y} = 0 \quad ; \quad \tau_{xy,x} + \sigma_{y,y} = 0$$

$$\varepsilon_x = u_{,x} \quad ; \quad \varepsilon_y = v_{,y} \quad ; \quad \gamma_{xy} = u_{,y} + v_{,x}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y \quad ; \quad \varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x \quad ; \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

cond. bordo esterno : $u = v = 0$

cond. bordo interno : $u = -c\alpha \sin\theta \quad ; \quad v = c\alpha\cos\theta$

cond. aggiuntiva :

$$\int_0^{2\pi}\left[-t_x c\sin\theta + t_y c\cos\theta\right]c\,d\theta = \int_0^{2\pi}\left[-(\sigma_x\cos\theta + \tau_{xy}\sin\theta)\sin\theta + (\tau_{xy}\cos\theta + \sigma_y\sin\theta)\cos\theta\right]c^2$$

$$= \int_0^{2\pi}\left[(\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)\right]c^2\,d\theta = M$$

1)
$$\sigma_{x,x} + \tau_{xy,y} = -\frac{2ay}{(x^2+y^2)^2} + \frac{2axy\cdot 4x}{(x^2+y^2)^3} - \frac{2by}{(x^2+y^2)^2} - \frac{b(x^2-y^2)}{(x^2+y^2)^3}\cdot 4y =$$

$$= \frac{2y}{(x^2+y^2)^3}\left[x^2(3a-3b) + y^2(b-a)\right] = 0 \quad \Rightarrow \quad \boxed{a = b}$$

$$\tau_{xy,x} + \sigma_{y,y} = \frac{2ax}{(x^2+y^2)^2} - \frac{2axy}{(x^2+y^2)^3}\cdot 4y + \frac{2bx}{(x^2+y^2)^2} - \frac{b(x^2-y^2)}{(x^2+y^2)^3}\cdot 4x =$$

$$= \frac{2x}{(x^2+y^2)^3}\left[x^2(a-b) + y^2(3b-3a)\right] = 0 \quad \Rightarrow \quad \boxed{a = b}$$

2)
$$\underline{t} = \underline{T}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \quad ; \quad x^2+y^2 = c^2 \quad ; \quad \frac{x}{c} = \cos\theta, \quad \frac{y}{c} = \sin\theta$$

$$\underline{t} = \frac{a}{c^2}\begin{pmatrix}-2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \\ \cos^2\theta - \sin^2\theta & 2\cos\theta\sin\theta\end{pmatrix}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} = \frac{a}{c^2}\begin{pmatrix}-\sin 2\theta\cos\theta + \cos 2\theta\sin\theta \\ \cos 2\theta\cos\theta + \sin 2\theta\sin\theta\end{pmatrix}$$

segue $\longrightarrow$

$$t_r = \underline{t} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \frac{a}{c^2}\left[-\cos^2\theta \sin 2\theta + \sin\theta\cos\theta\cos 2\theta + \sin\theta\cos\theta\cos 2\theta + \sin^2\theta\sin 2\theta\right] = 0$$

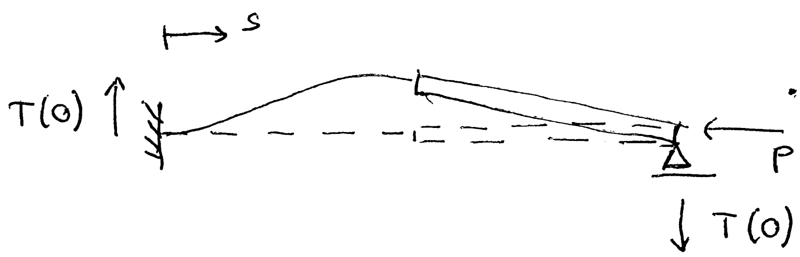
$$t_\theta = \underline{t} \cdot \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} = \frac{a}{c^2}\left[\sin\theta\cos\theta\sin 2\theta - \sin^2\theta\,\cancel{\cos\theta}\cos 2\theta + \cos^2\theta\cos 2\theta + \sin\theta\cos\theta\sin 2\theta\right] =$$

$$= \frac{a}{c^2}\left[\sin^2 2\theta + \cos^2 2\theta\right] = \frac{a}{c^2}$$

$$\int_0^{2\pi} t_\theta \cdot c \cdot (c\,d\theta) = a \cdot 2\pi = M \Rightarrow \quad a = \frac{M}{2\pi} \quad (= b)$$

4) $\dfrac{\sigma_x}{\sigma_y} = -1$ ma, ad es., sul lato CD $\varepsilon_x = 0 \Rightarrow \dfrac{\sigma_x}{\sigma_y} = \nu$

Problema 2



$v(0) = 0 \quad ; \quad v'(0) = 0$

$v(L) = -L\,v'(L) \quad ; \qquad -EJ\,v'''(L) = -EJ\,v'''(0) + P\,v'(L)$