Università degli Studi di PISA
Dipartimento di Ingegneria Strutturale

Pisa, 9 Ottobre 2006

Un’applicazione pratica del modello di frattura (rivisitato) di Bourdin, Francfort e Marigo: lo studio del degrado nel Panthéon Francese a Parigi

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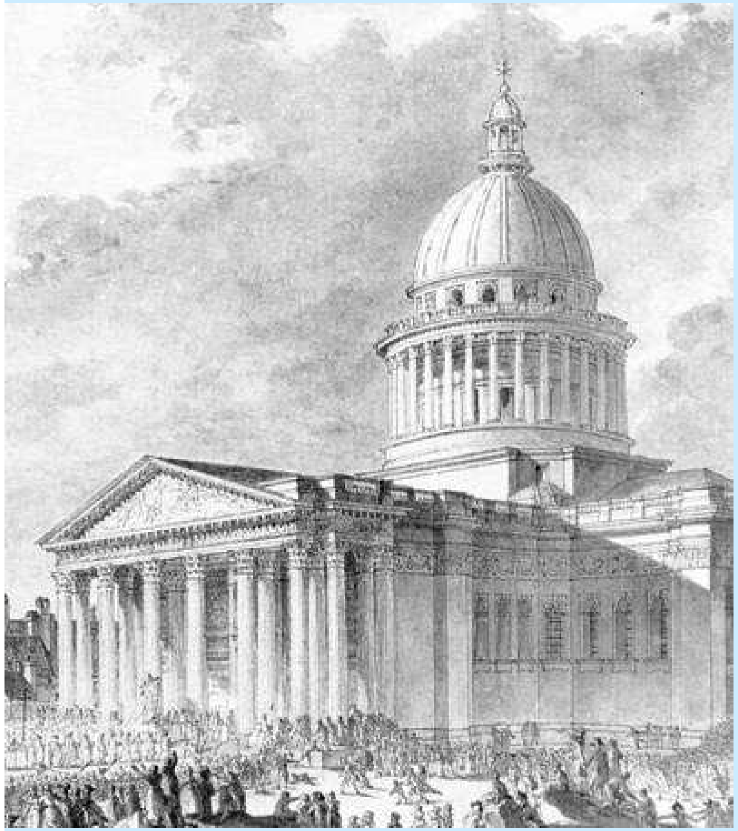
Il lavoro è stato svolto in collaborazione con **Giovanni Lancioni**, dell’Università Politecnica delle Marche.

Studio nell’ambito di uno programma di ricerca commissionato dal Ministero Francese della Cultura e della Comunicazione, coordinato dal Prof. Arch. **Carlo Blasi**, dell’Università di Parma.


AUX GRANDS HOMMES LA PATRIE RECONNAISSANTE
2005 06 02



1754 Luigi XV supports the construction of the new Church dedicated to Sainte Geneviève;

1755 J.G. Soufflot is charged with the design;

1757 J-G. Soufflot publishes the design, which contemplates the use of "**pierre armée**";

1758 the construction work starts;

1764 ceremony for the laying of the foundation stone;

1768-70 Pierre Patte questions about the **stability**;

1776 First cracks in the pylons of the main dome;

1780 Soufflot dies and his assistant, J. B. Rondelet, succeeds in the work direction;

1790 Rondelet completes the main dome;

1791 Quatremère de Quincy is charged to convert the church into Panthéon;

1793 Quatremère bricks in the windows and demolishes the bell towers;

1797 Rondelet publishes his "Mémoire historique sur le dôme du Pantèon français;

1798 The pylons of the main dome are shored up;

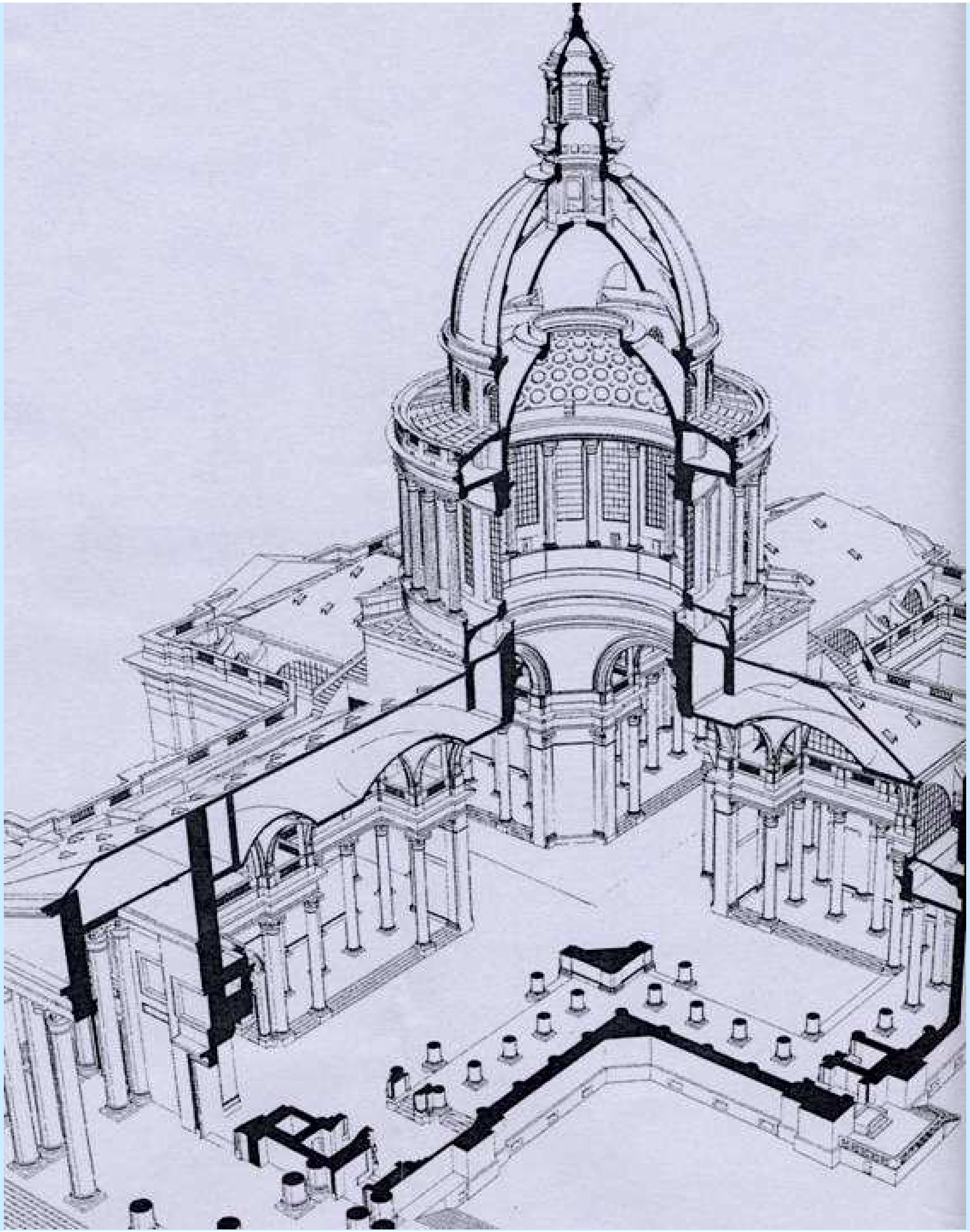
1806-12 Rondelet consolidates the pylons; cracks form;

…….some blocks fracture and big stone fragments fall down from vaults and arches;

1985 The **access** to the monument is **interdicted**;

2005 The monument is opened again.

The structural scheme

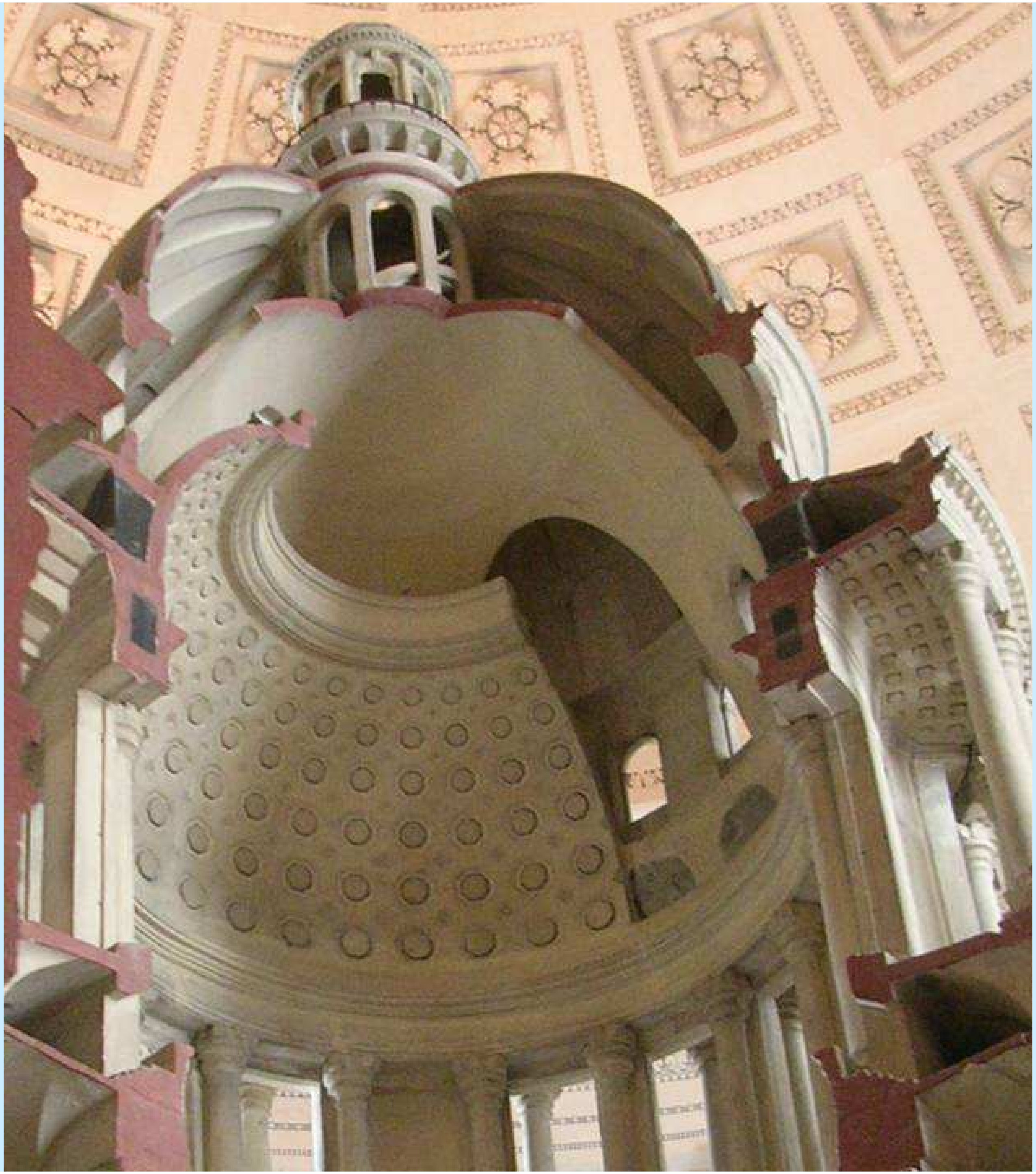


Dome

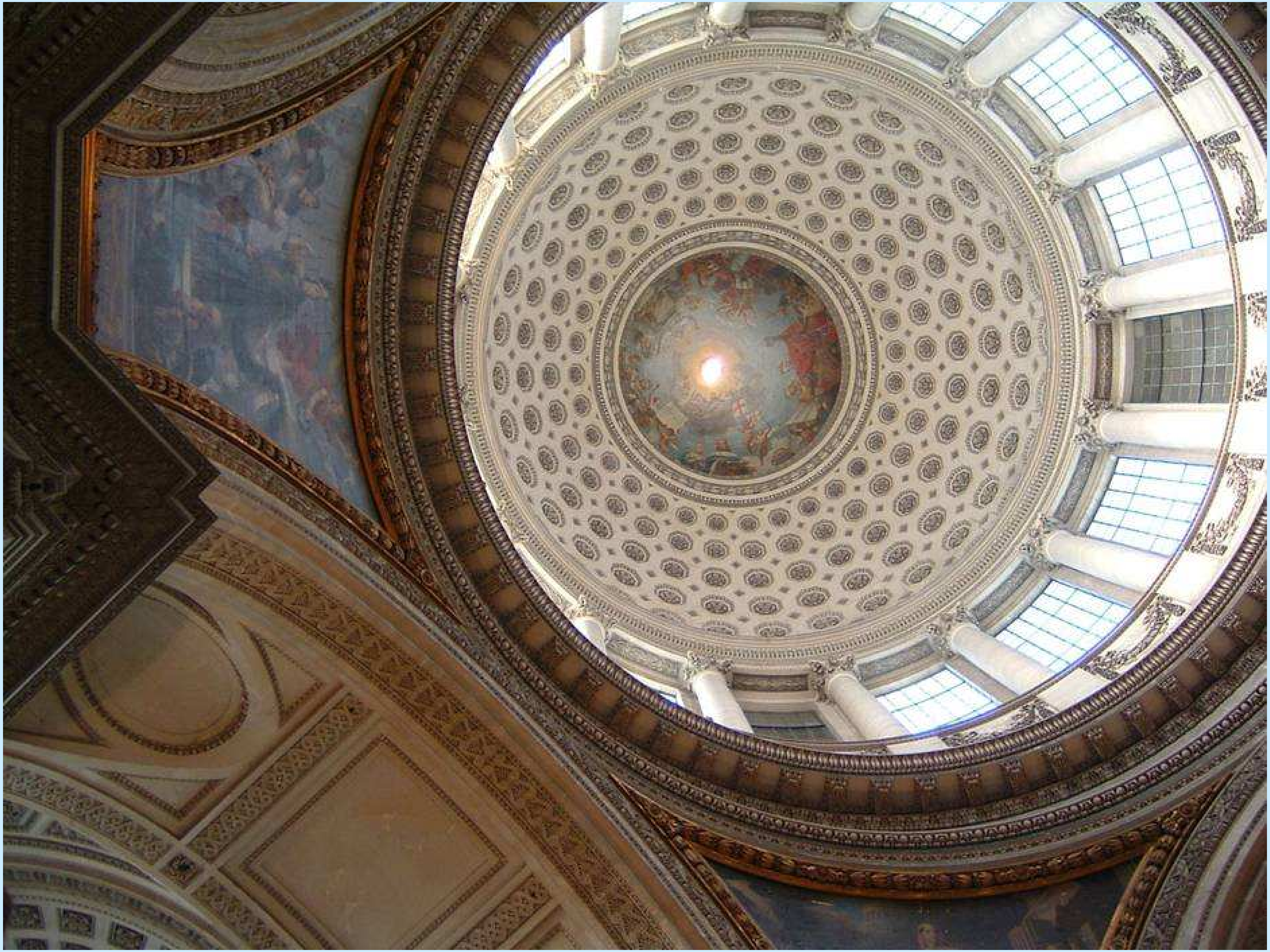
Tambour

Plafond

Very thin structure in “reinforced stone”



THREE DOMES

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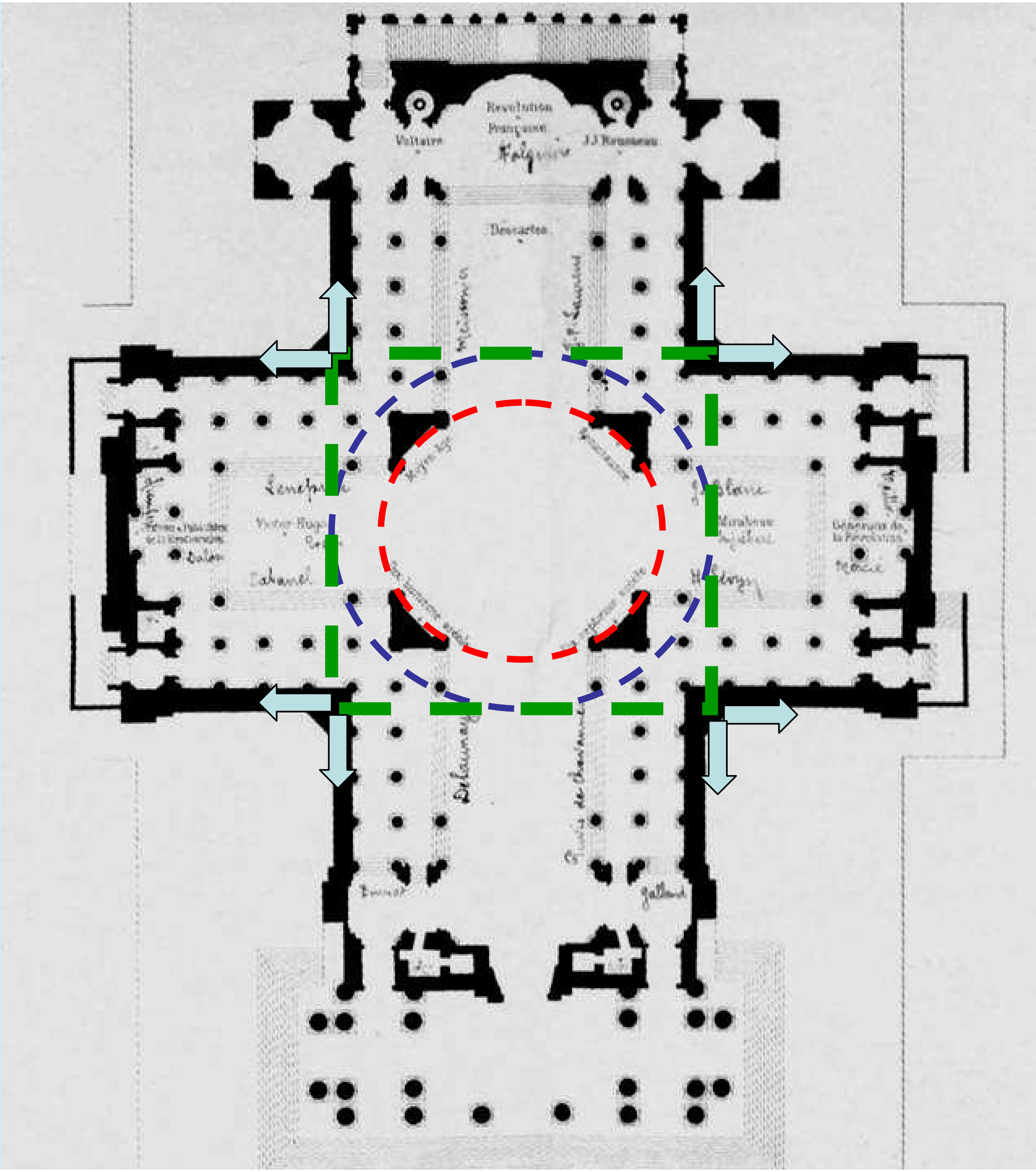
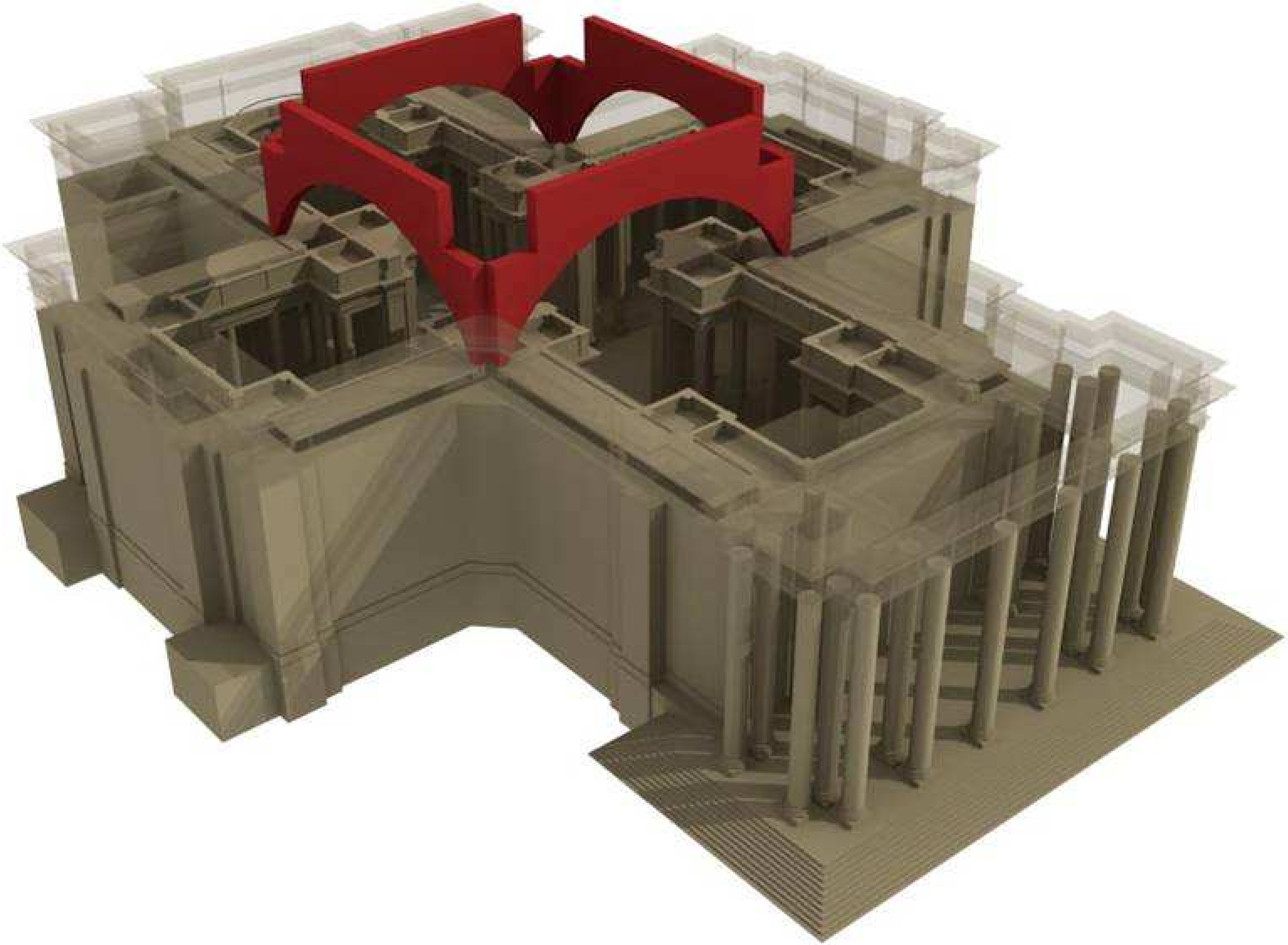


Revolution Française
Voltaire
J.J. Rousseau
Descartes
Victor Hugo

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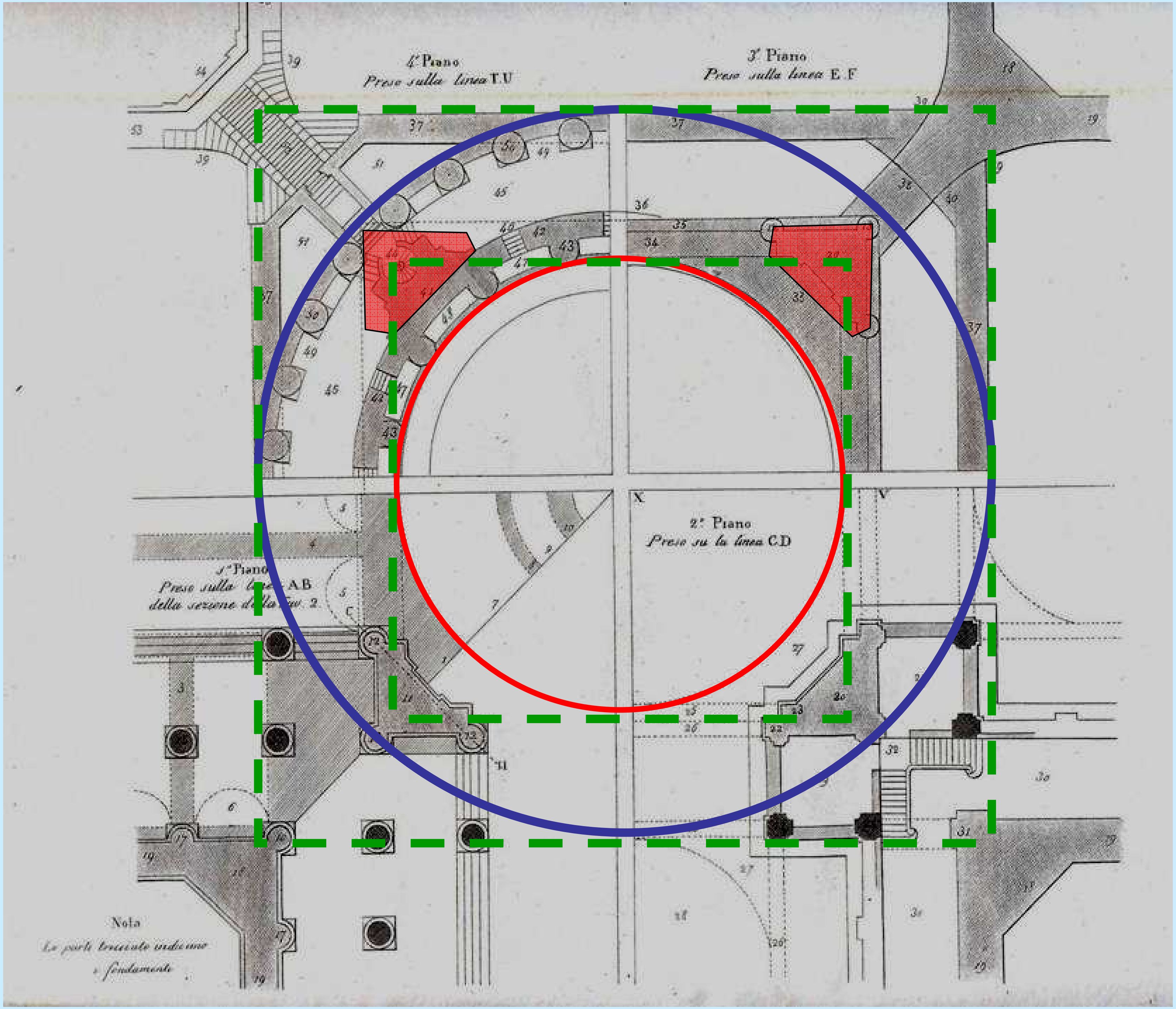
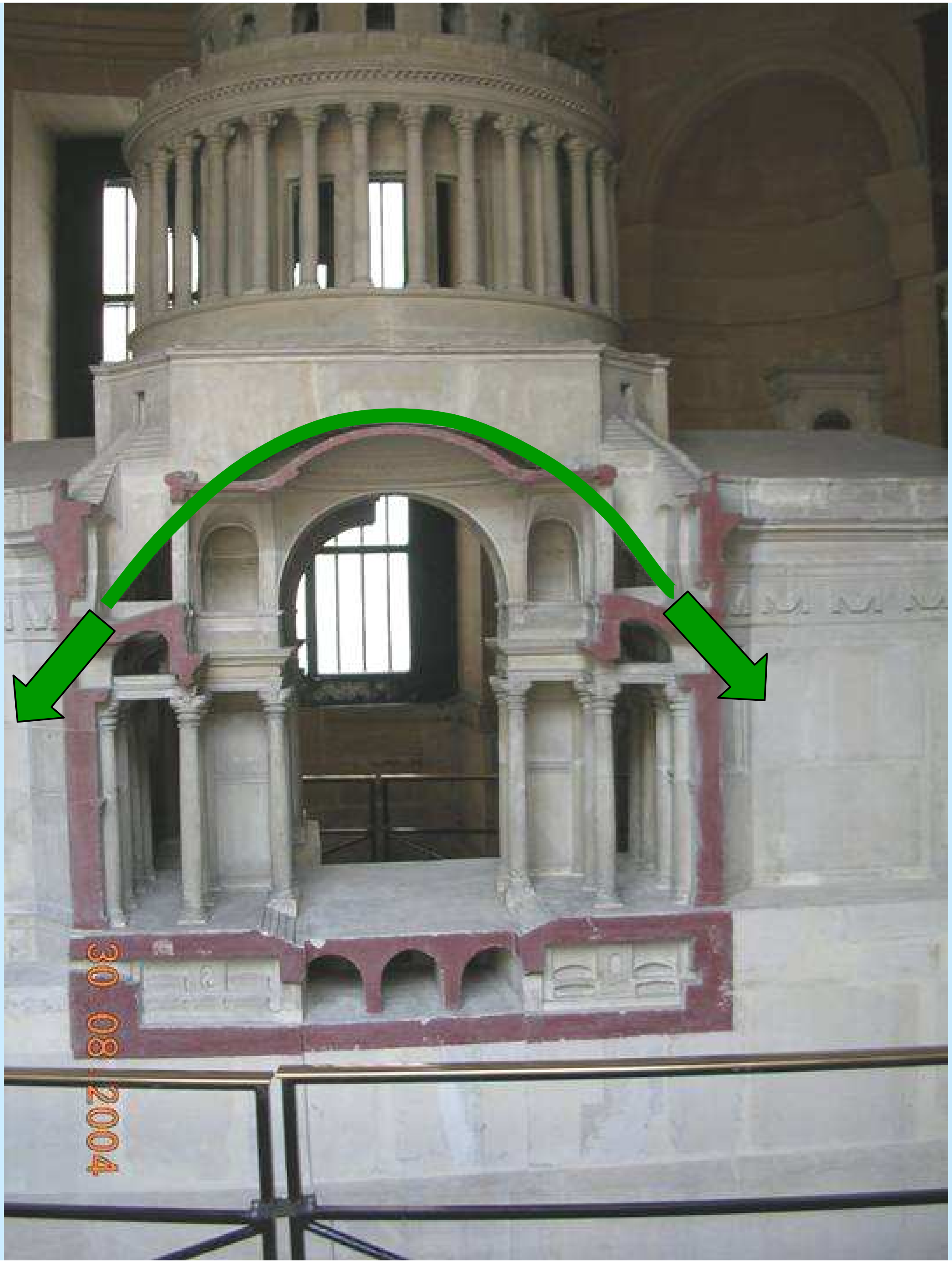


4° Piano
Preso sulla linea T.U
3° Piano
Preso sulla linea E.F
2° Piano
Preso su la linea C.D
1° Piano
Preso sulla linea A.B
della sezione della tav. 2
Nota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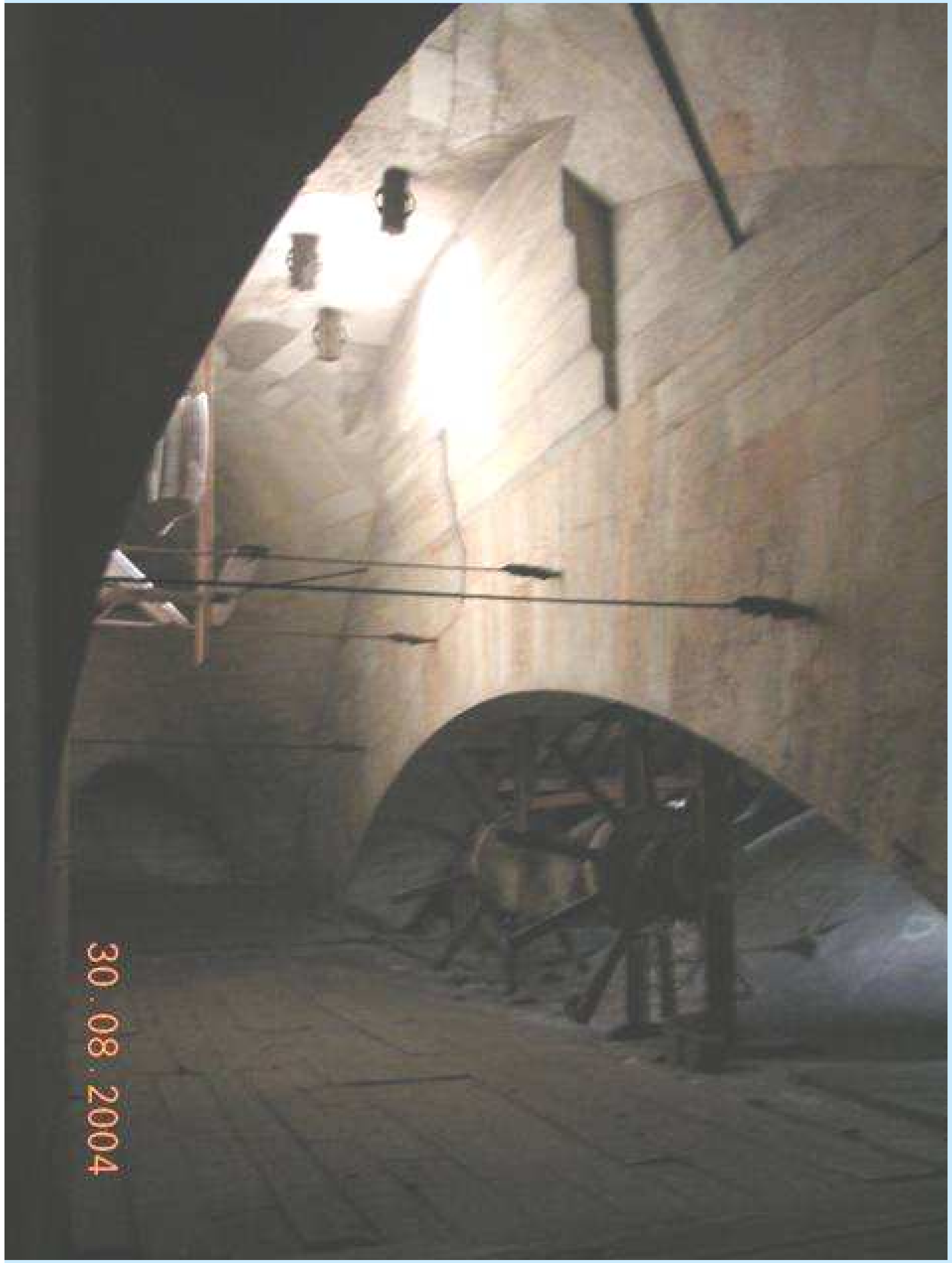
Arch independent
from the vaults
30.08.2004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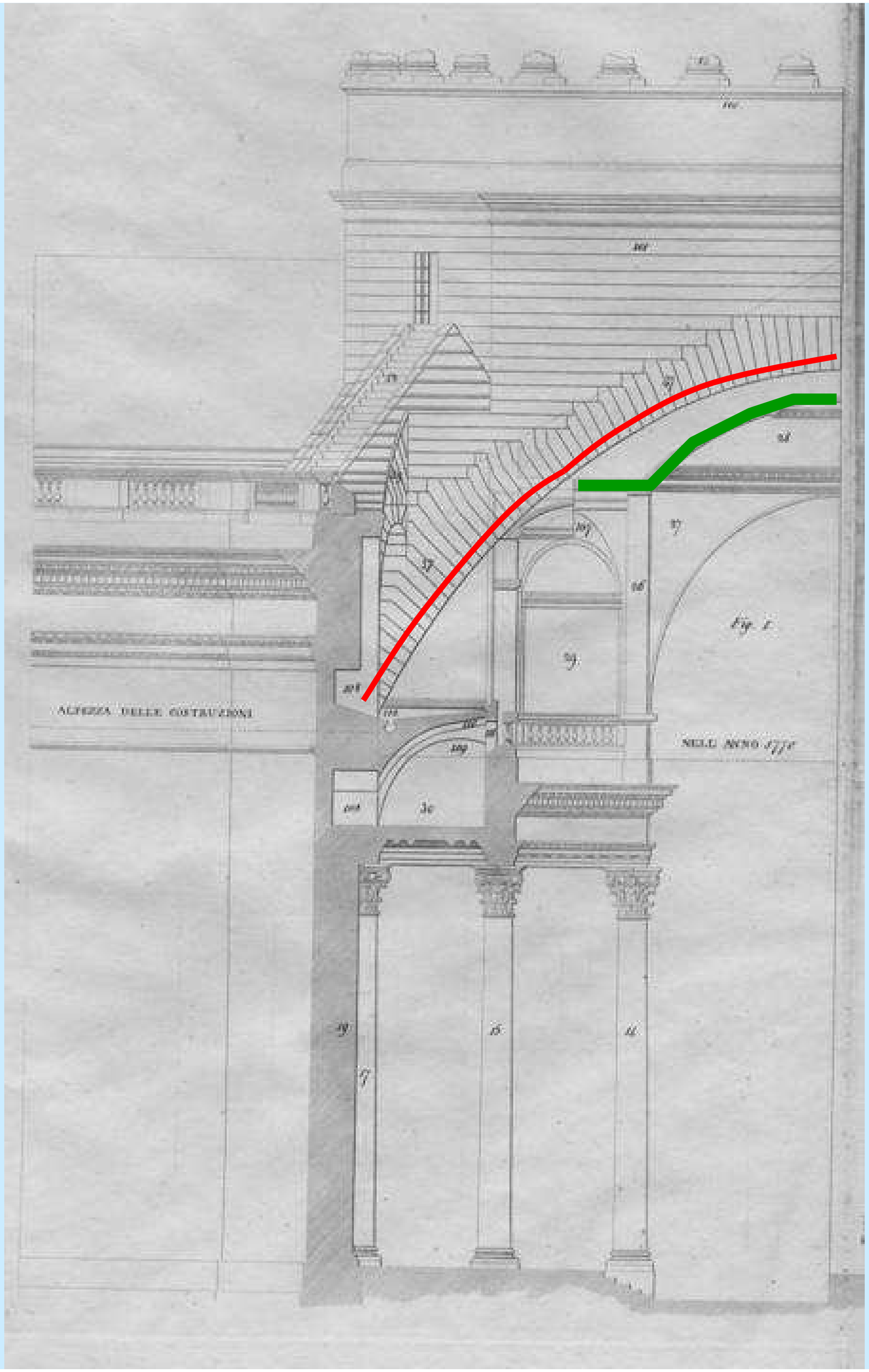
ALTEZZA DELLE COSTRUZIONI
NELL ANNO
Fig. 1


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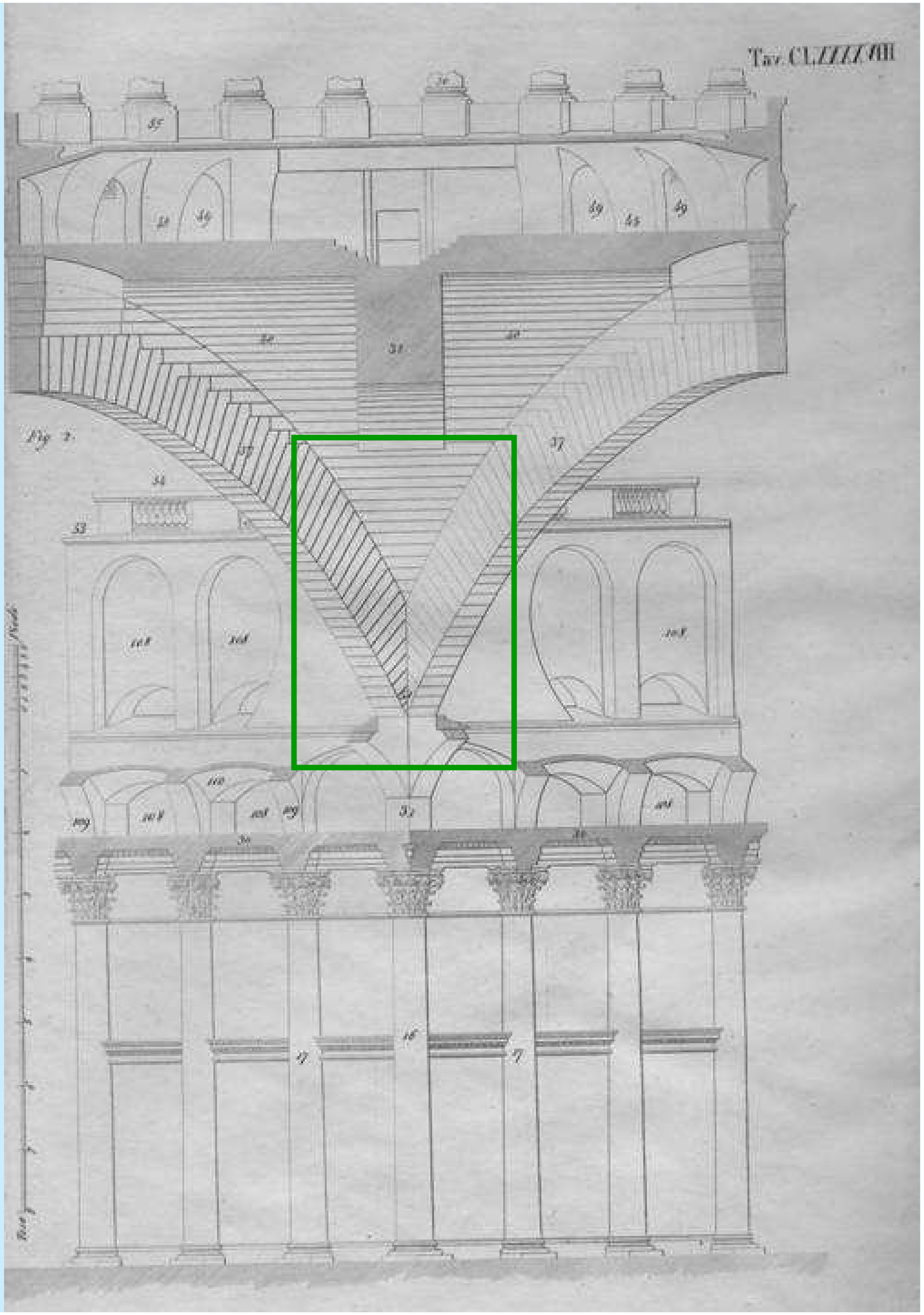
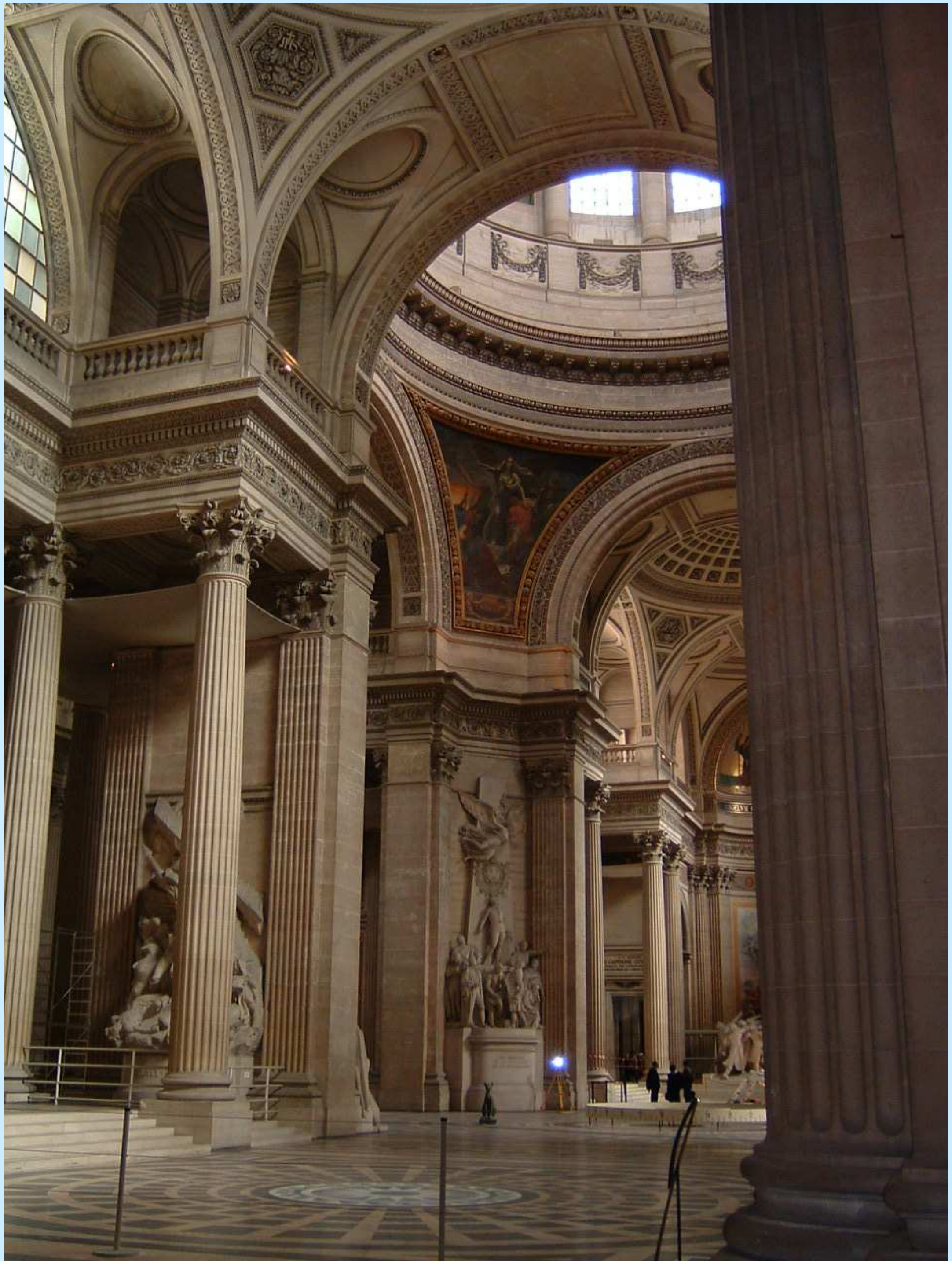
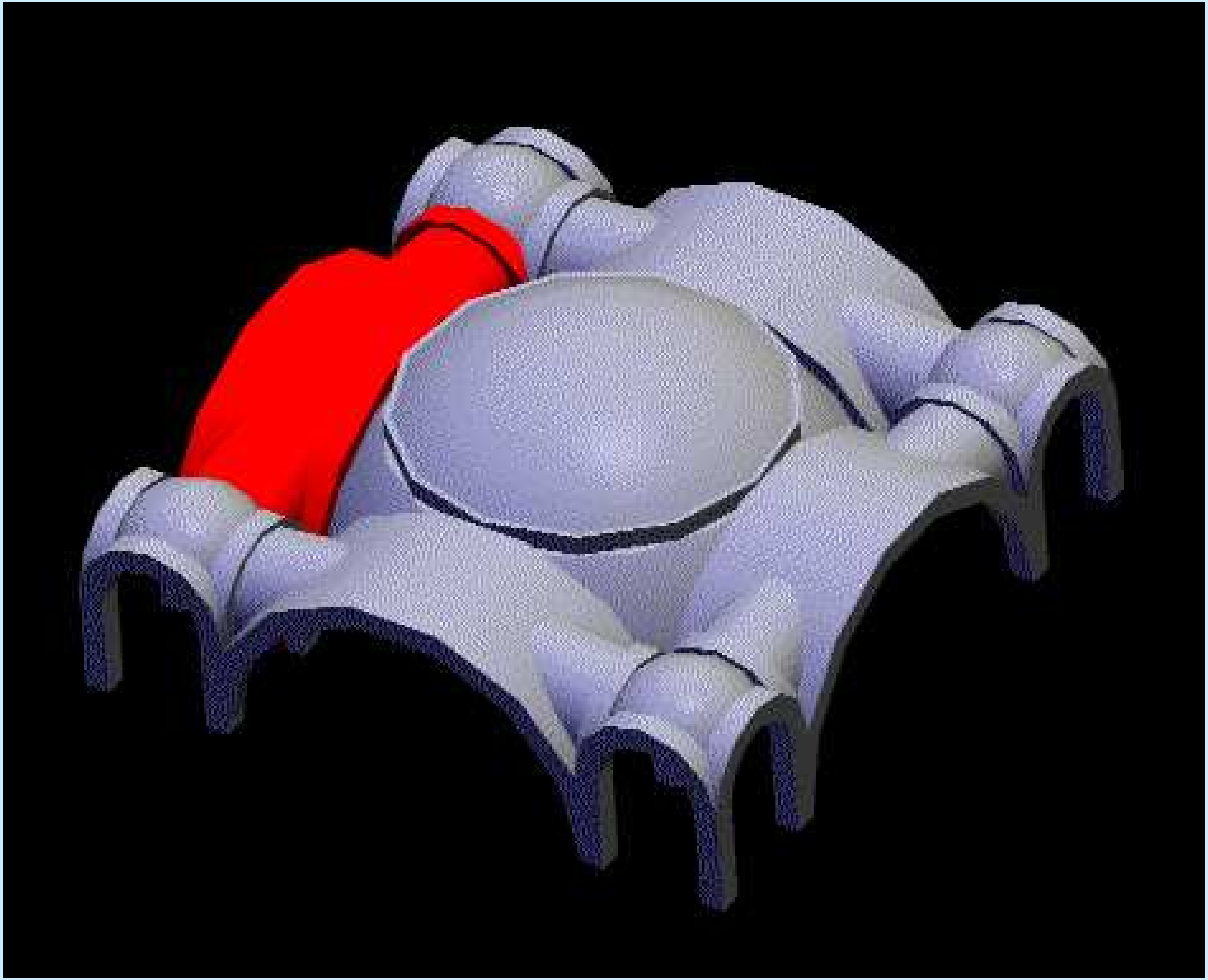
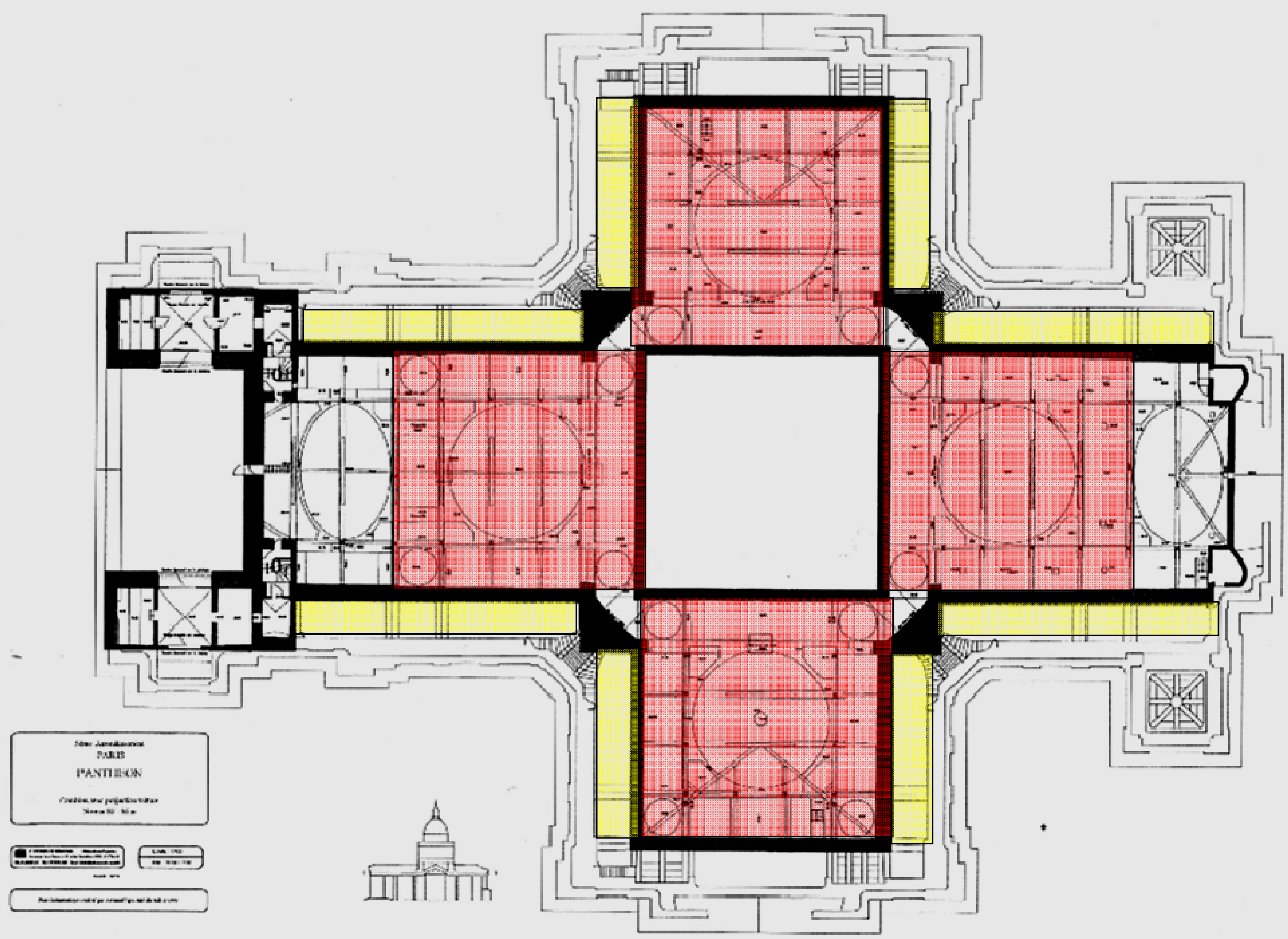
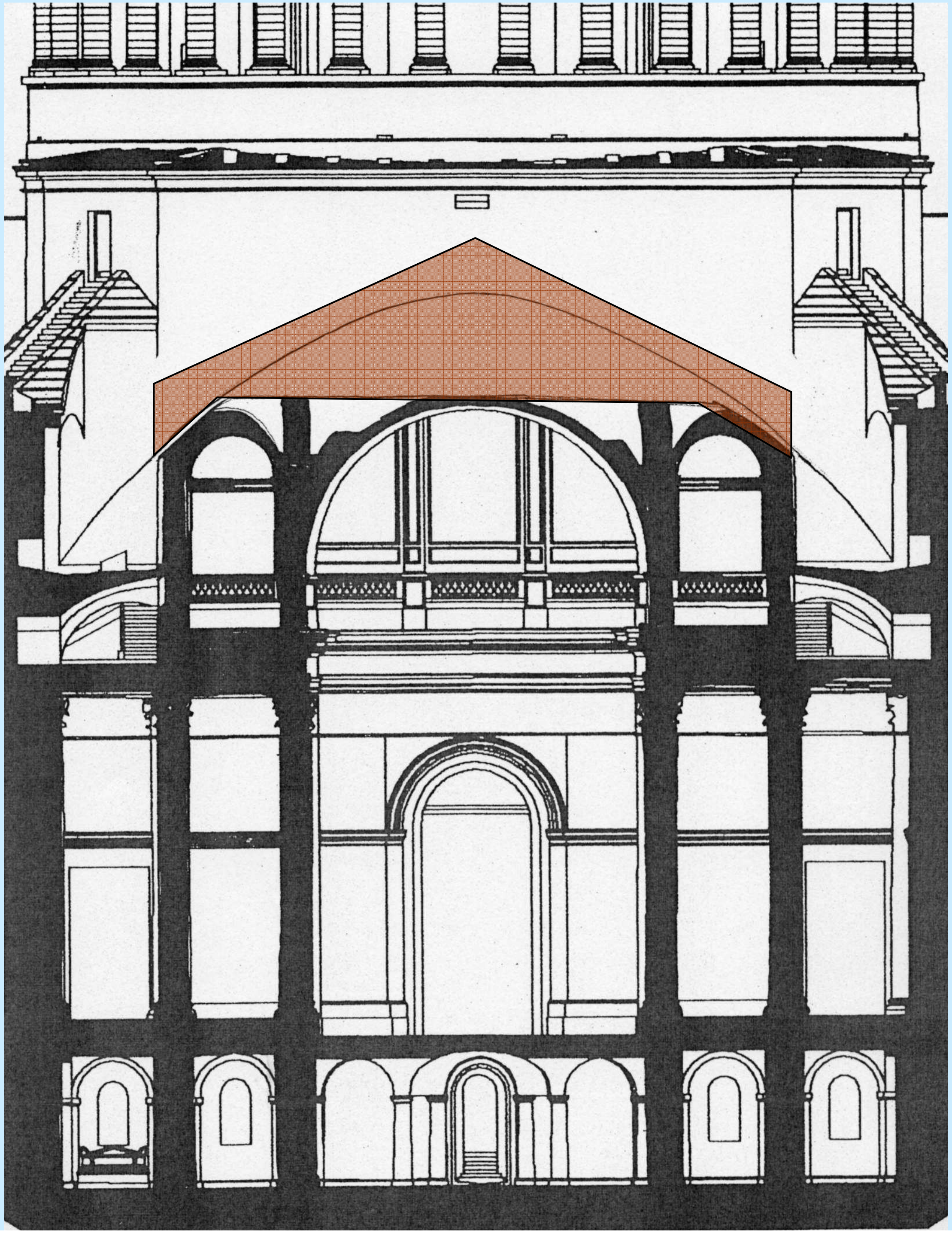

Tav. CLXXXVIII
Fig. 2






PARIS
PANTHEON



The construction material

pierre armée

(reinforced Stone)

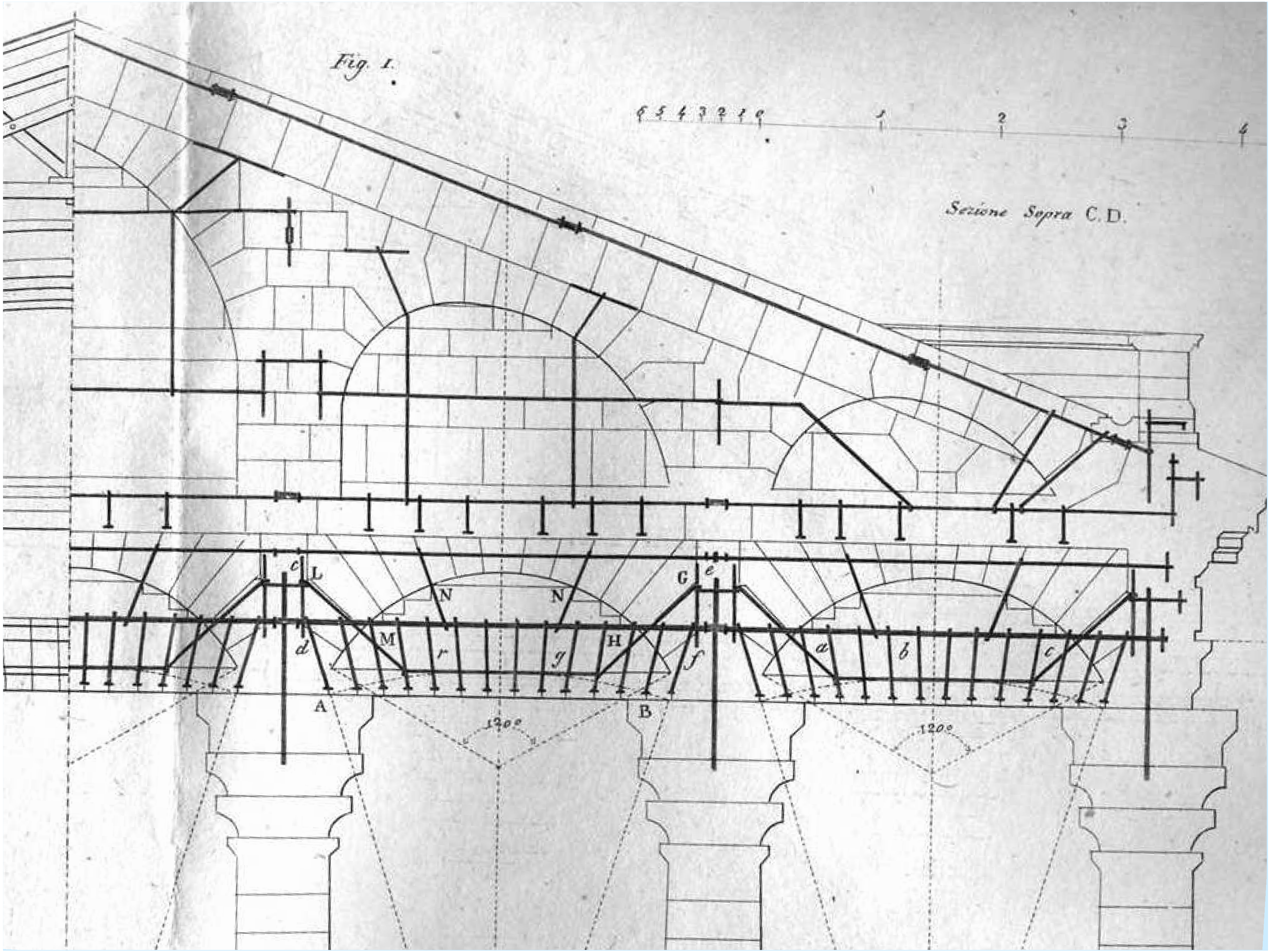
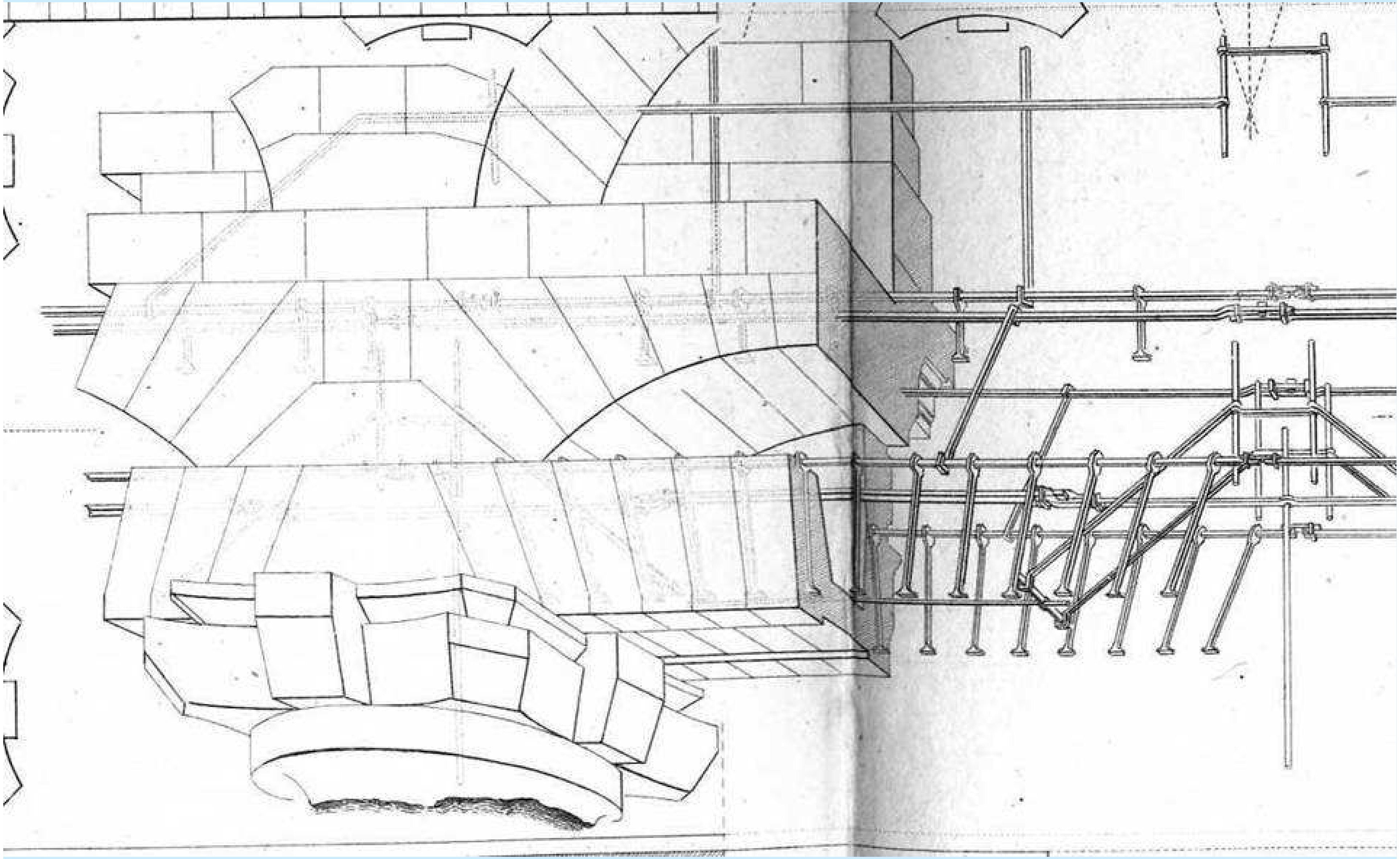
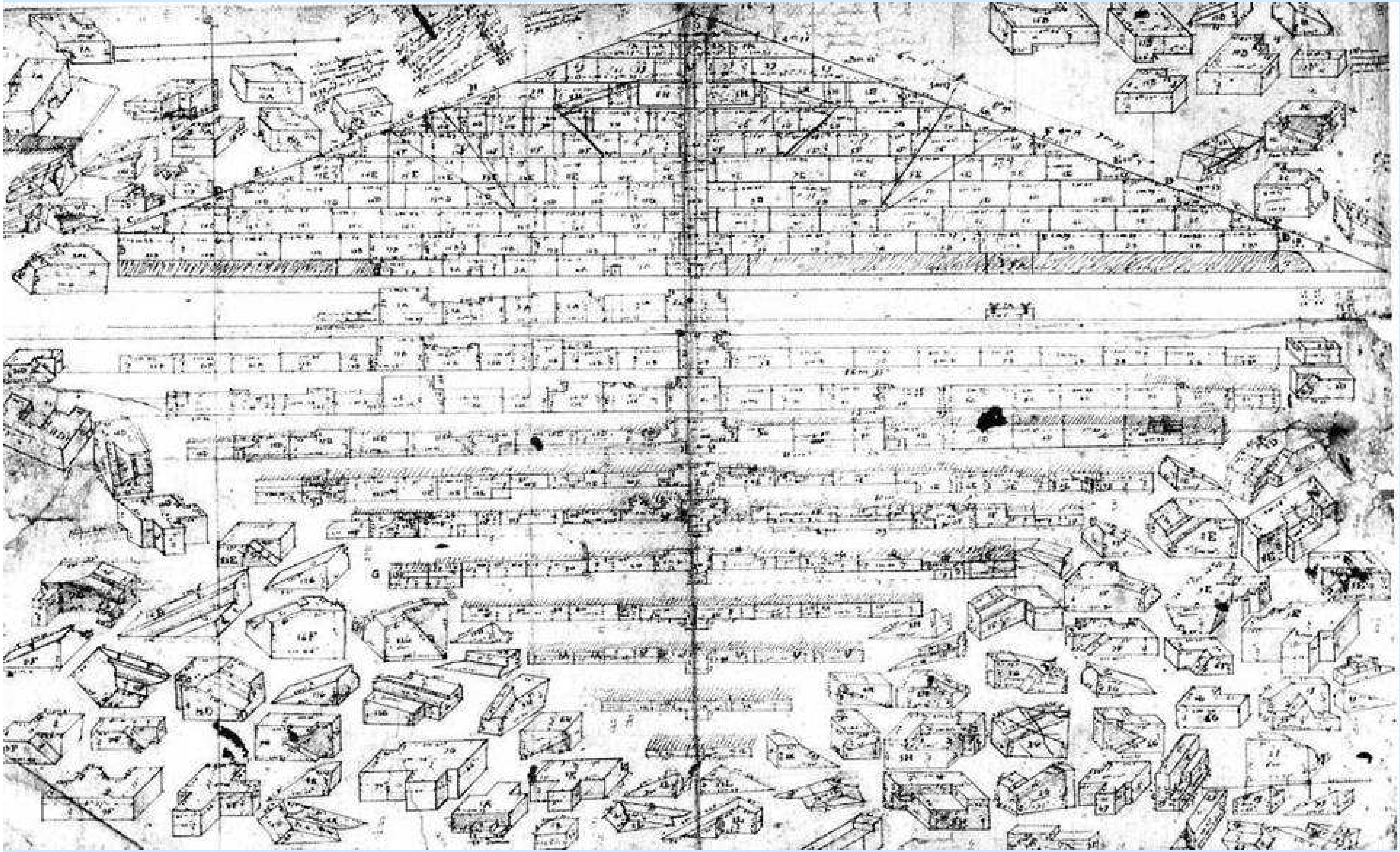
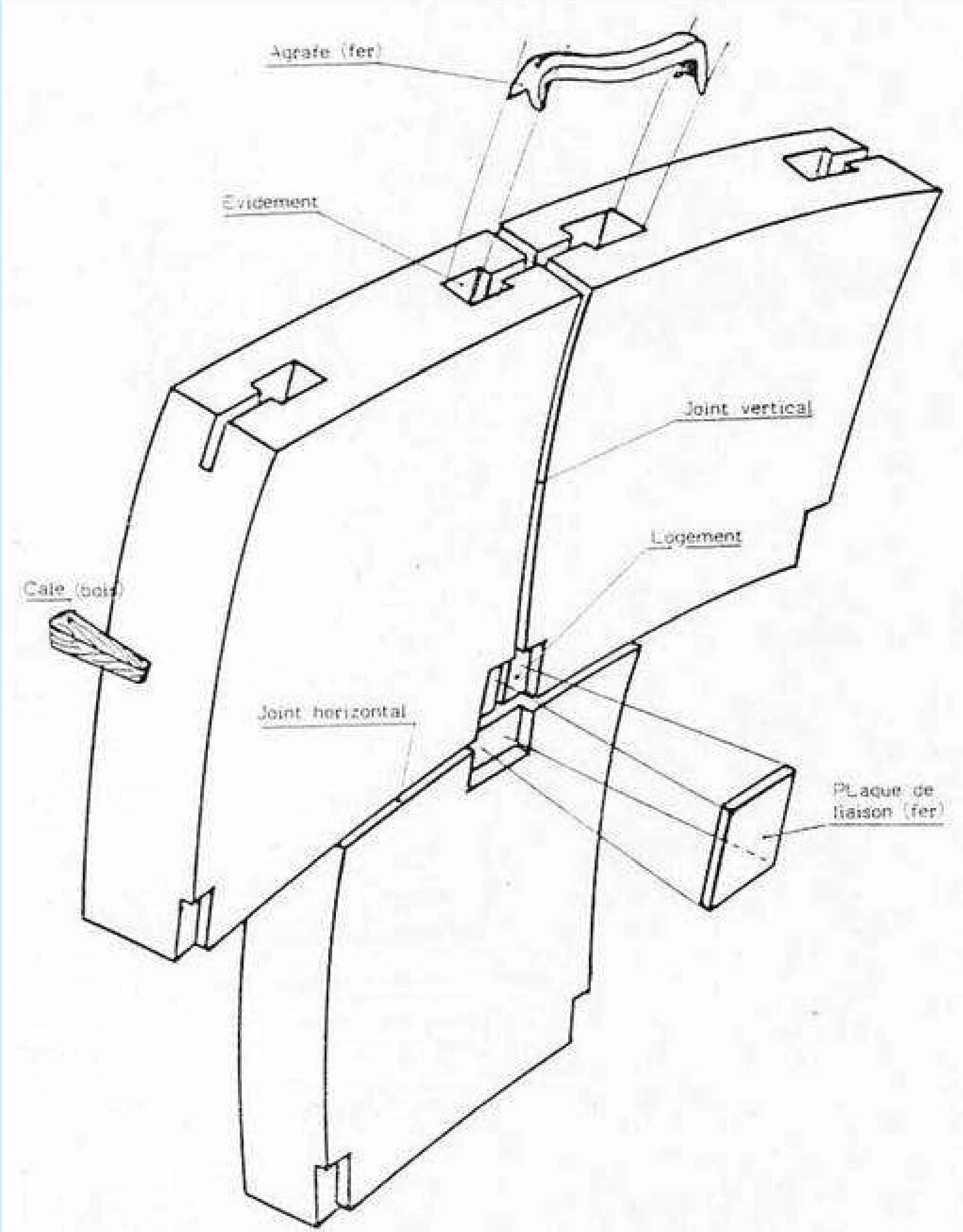
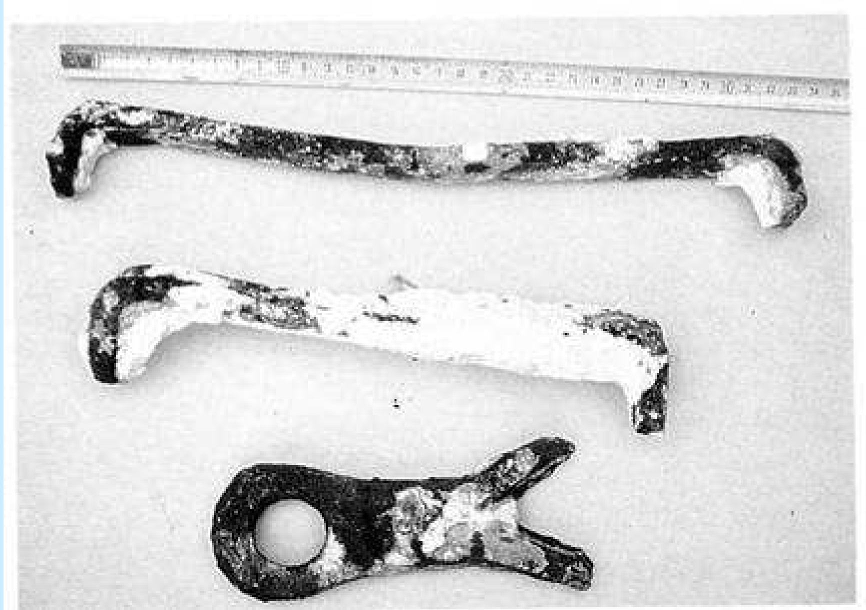


Fig. I.
Sezione Sopra C.D.
6 5 4 3 2 1 0
1
2
3
4
A
B
1200
1200





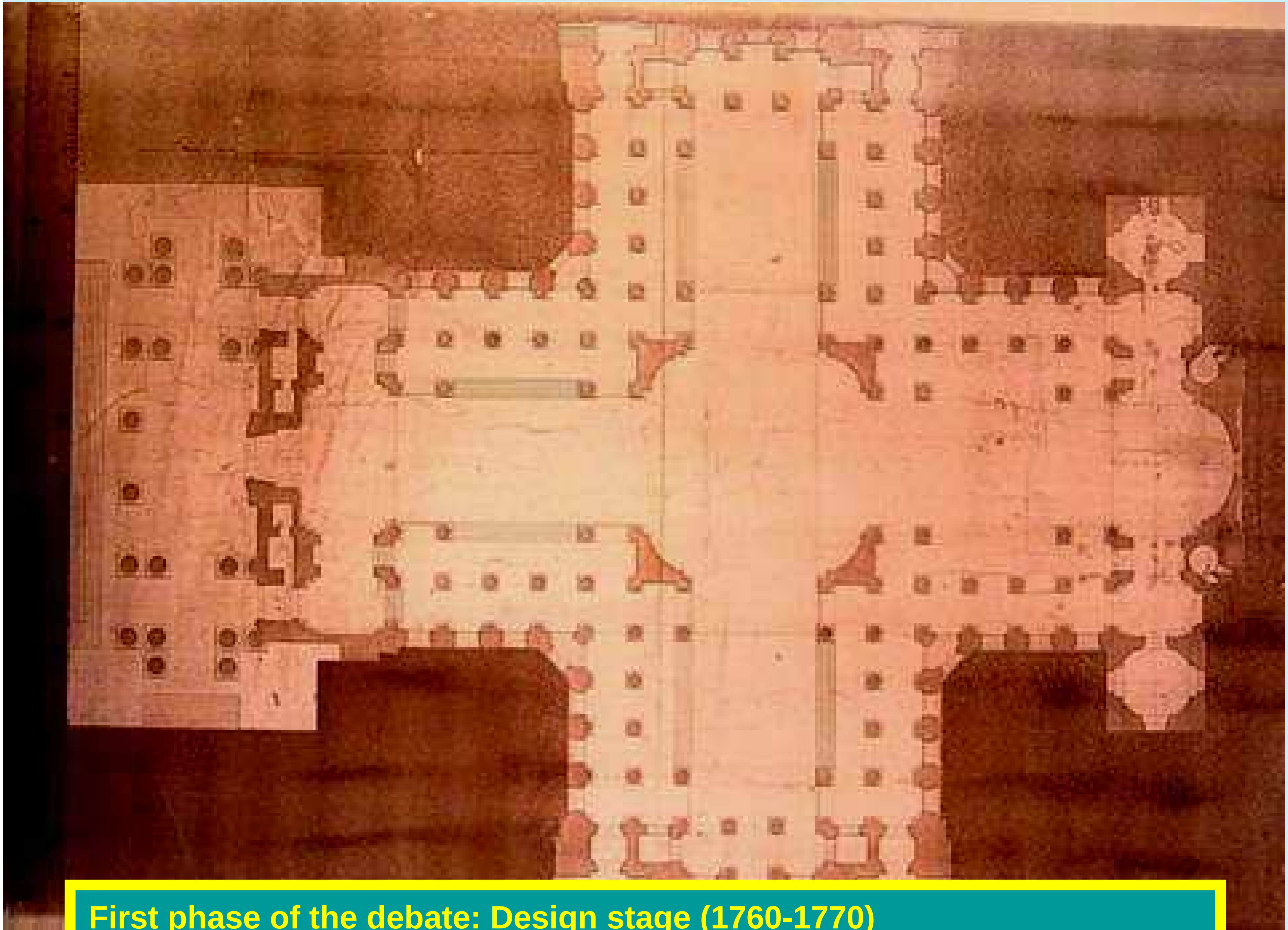




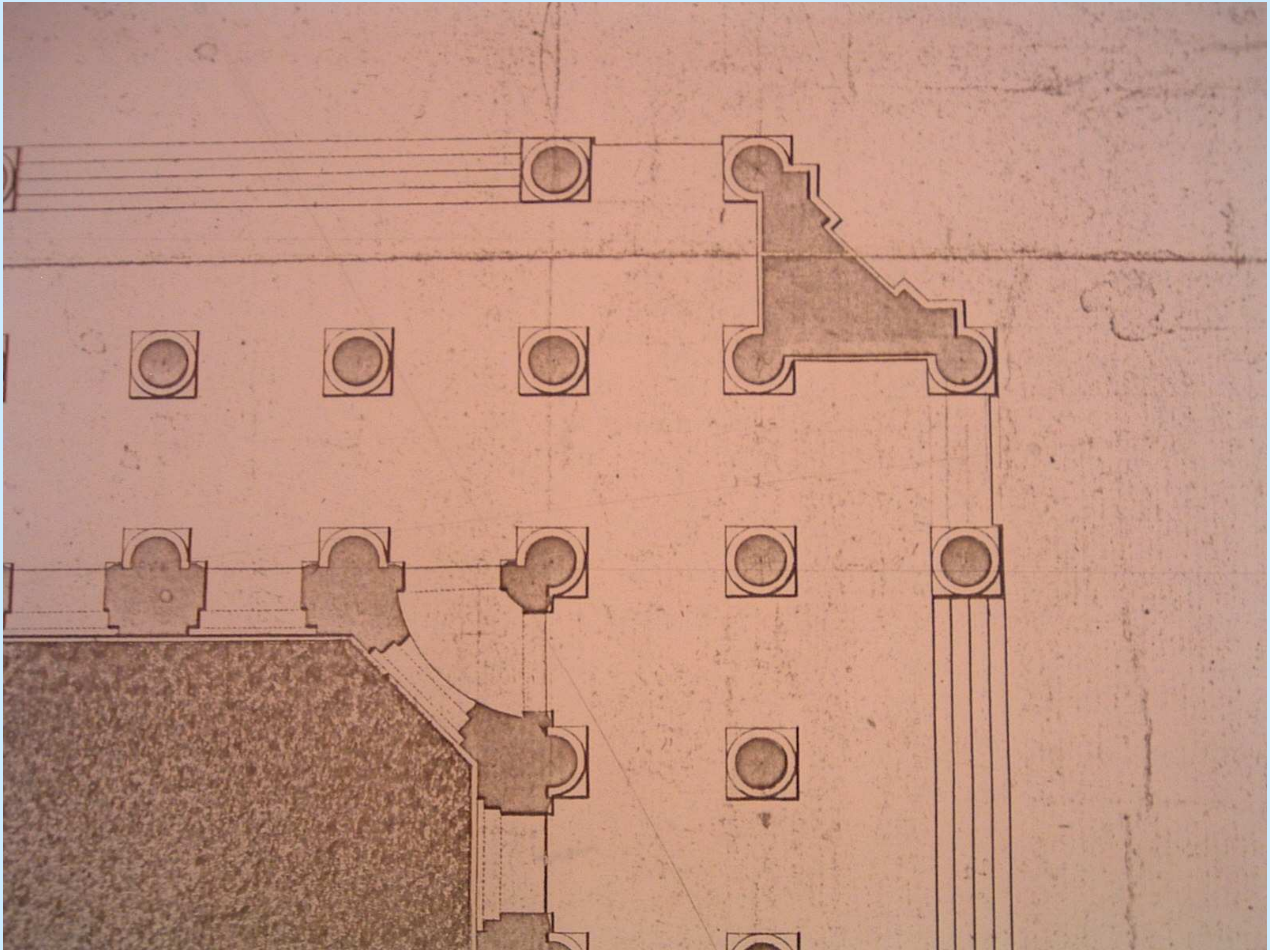
Steel reinforcement of the stone ashlars

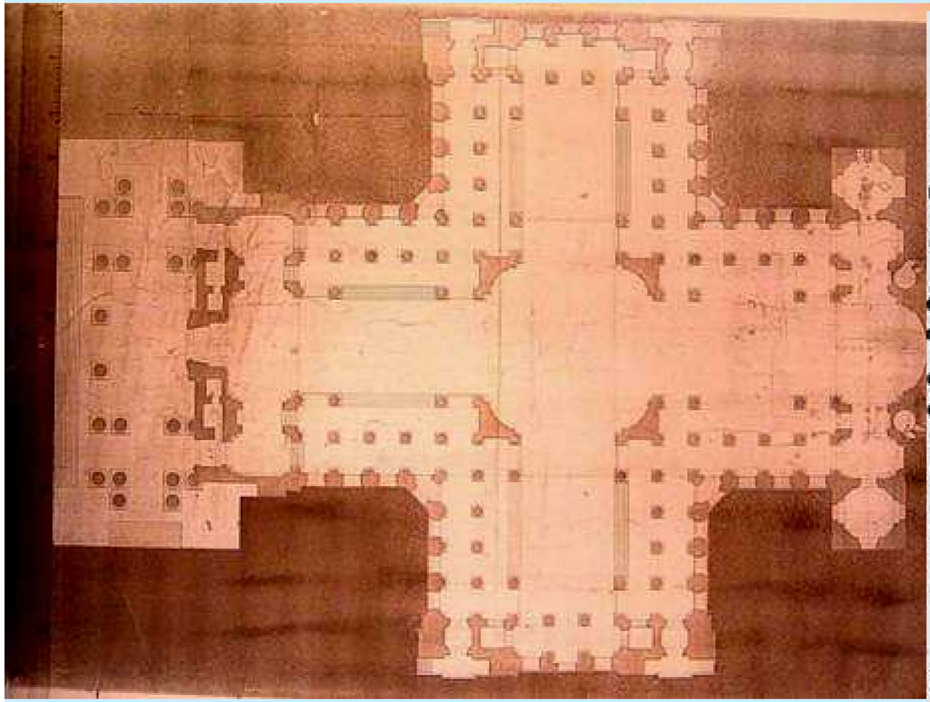
Main faults and debates

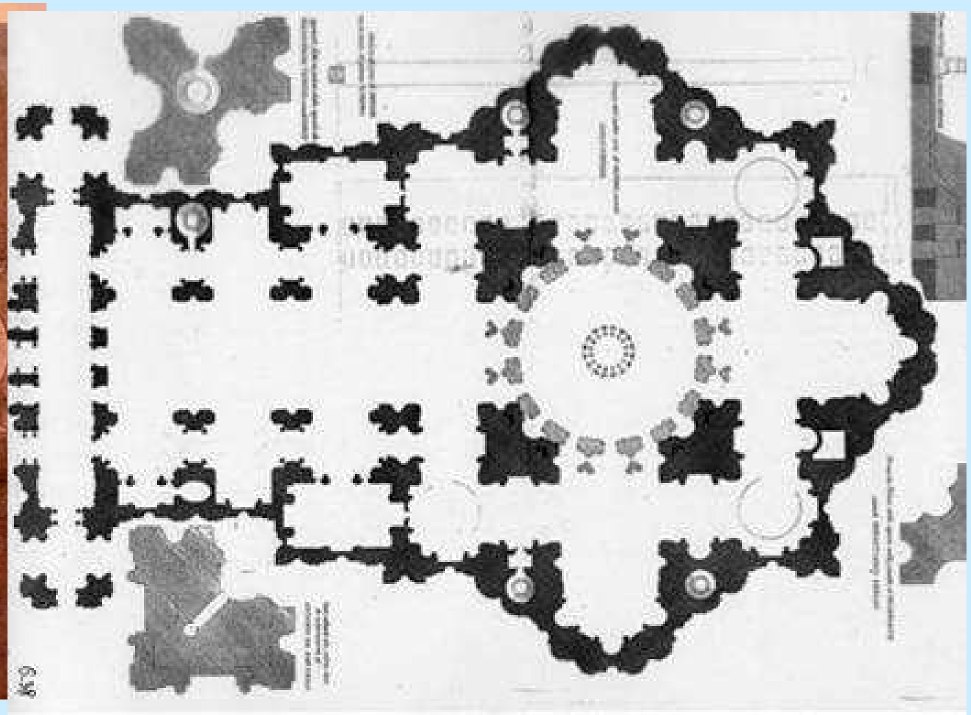
Three phases

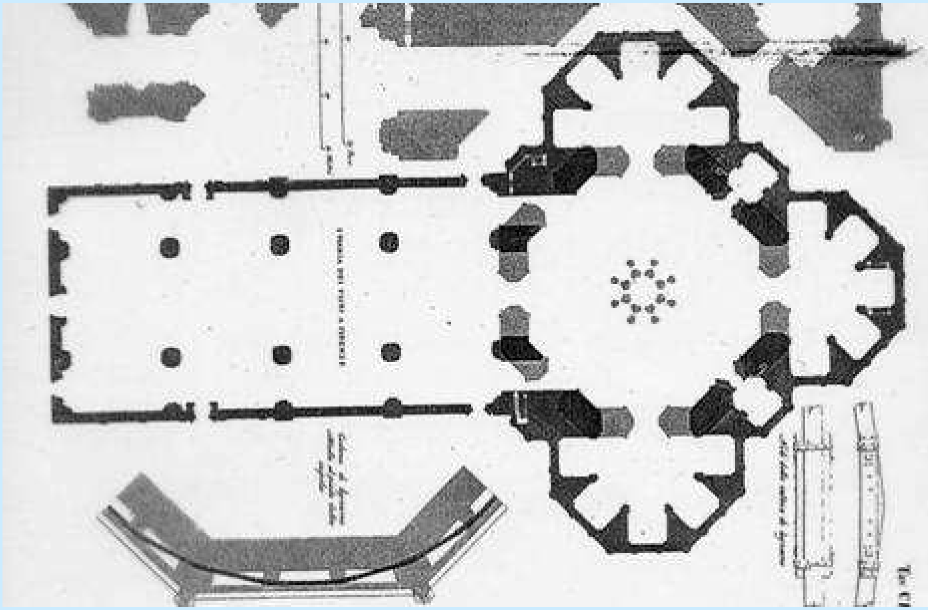


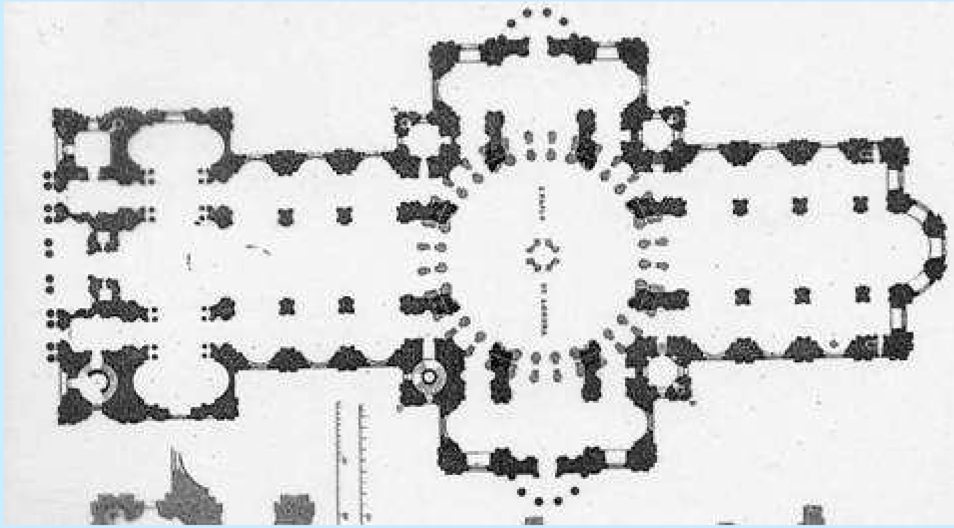
First phase of the debate: Design stage (1760-1770)

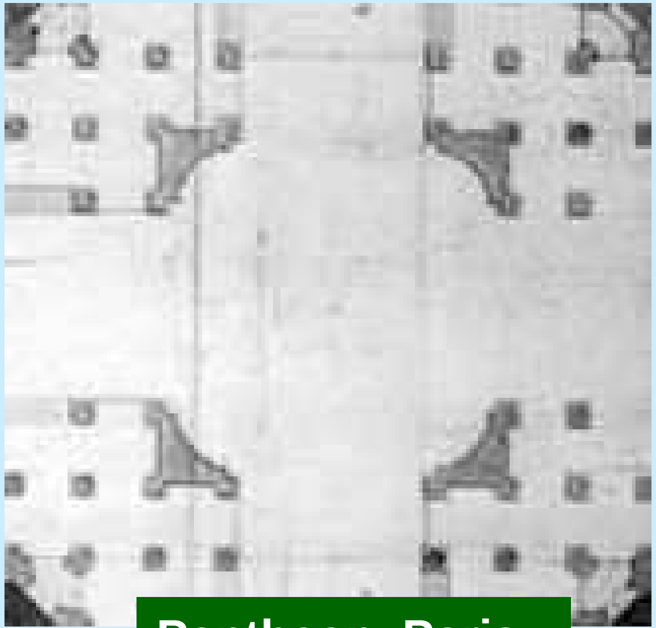






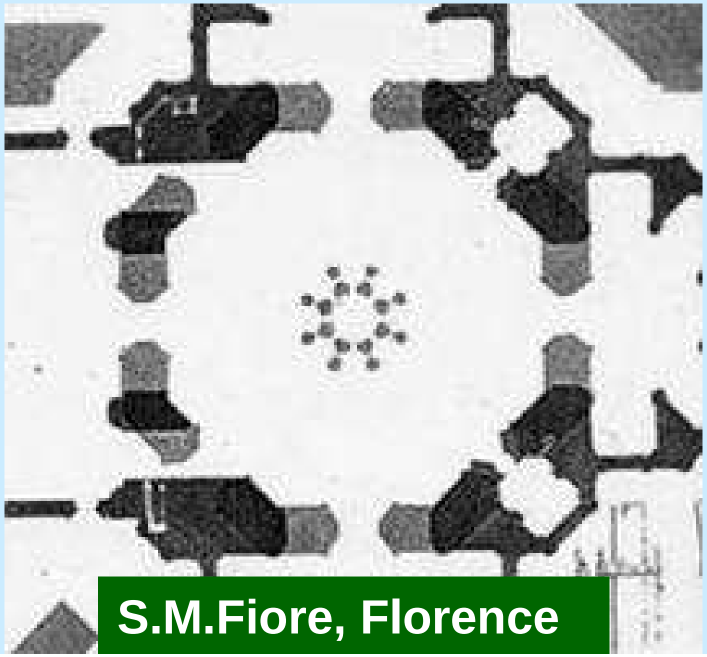




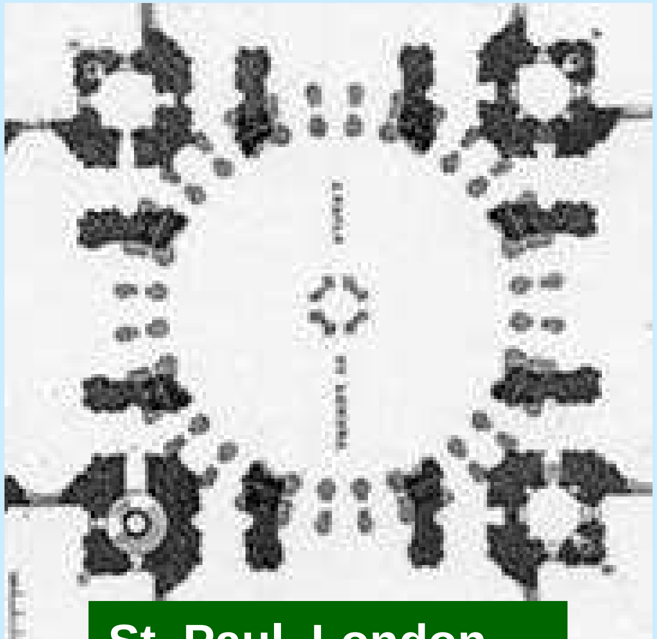


Pantheon, Paris

St. Peter, Rome



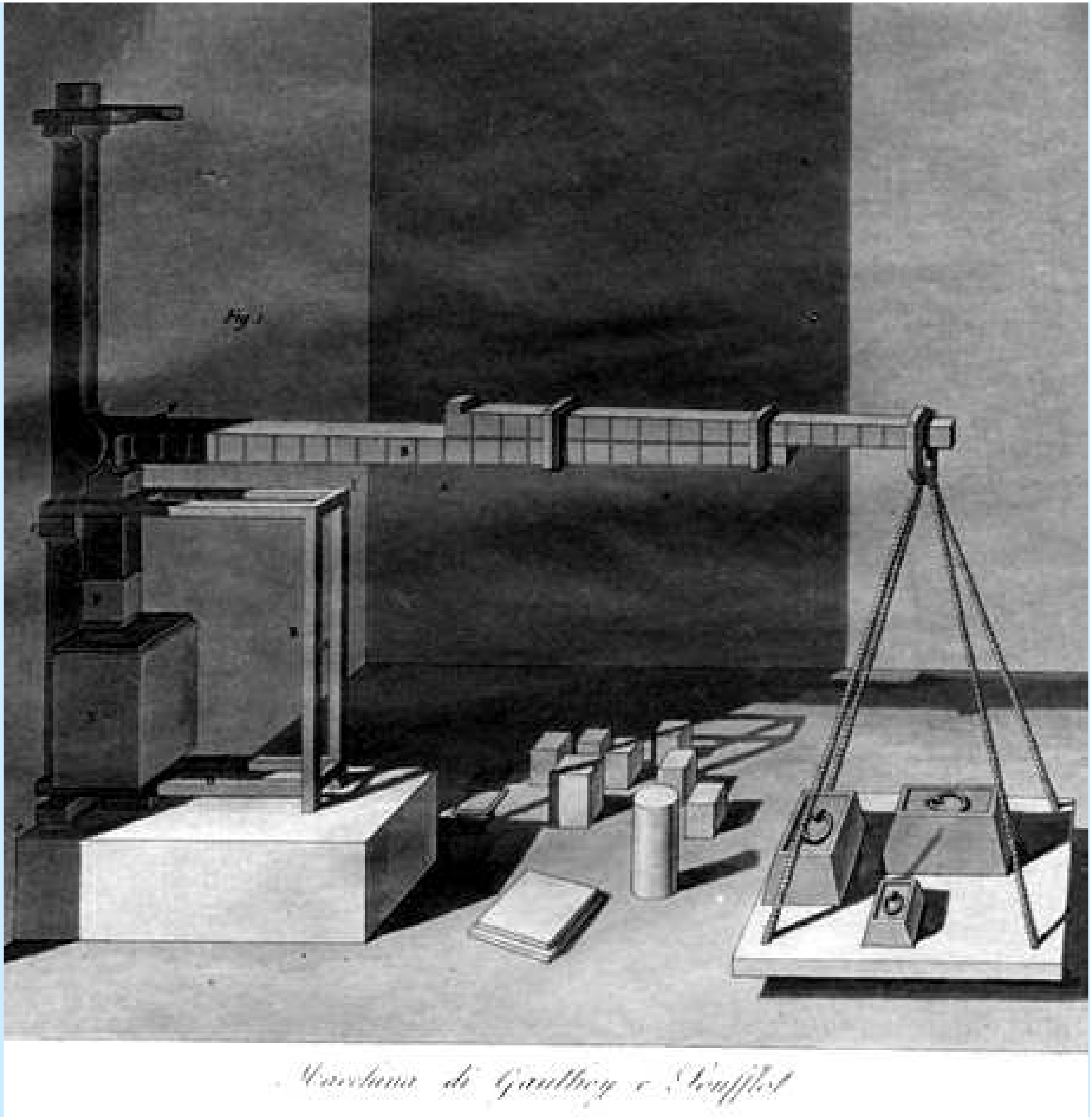
S.M.Fiore, Florence



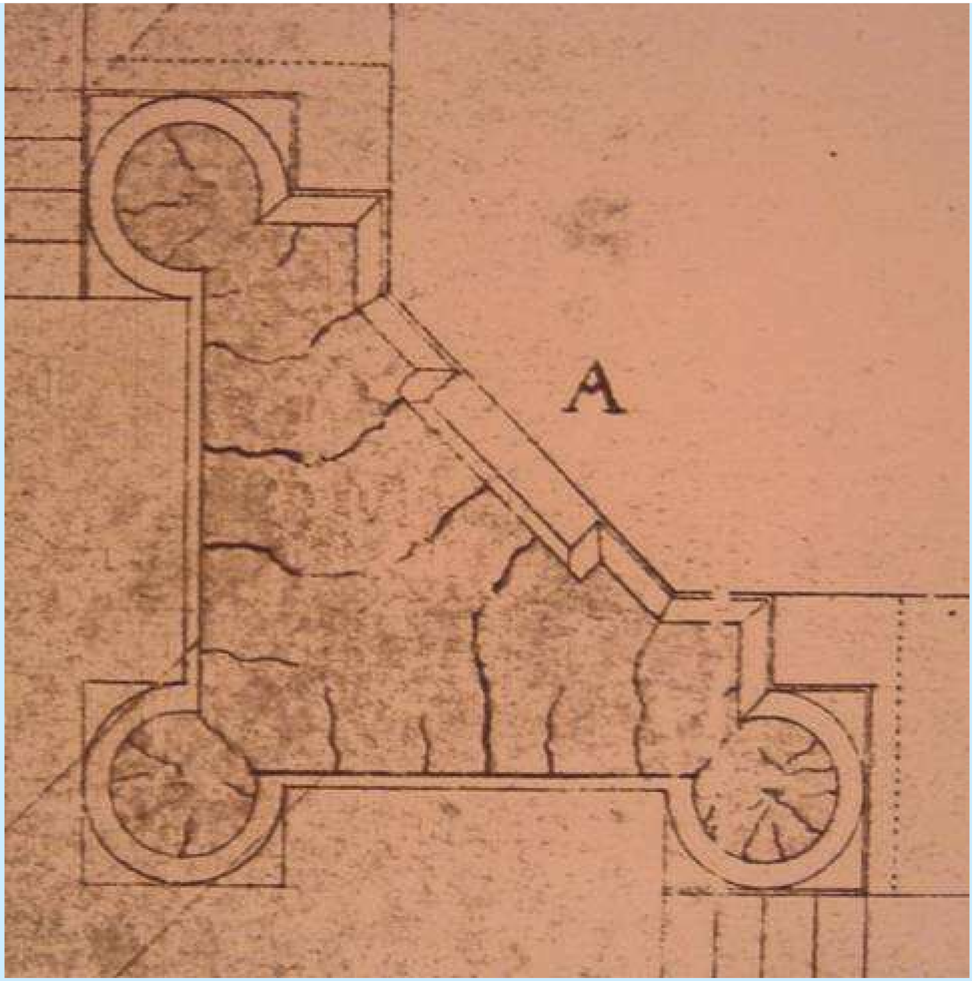
St. Paul, London

MATERIAL TESTING, STRESS CALCULATION AND STRENGTH LIMIT

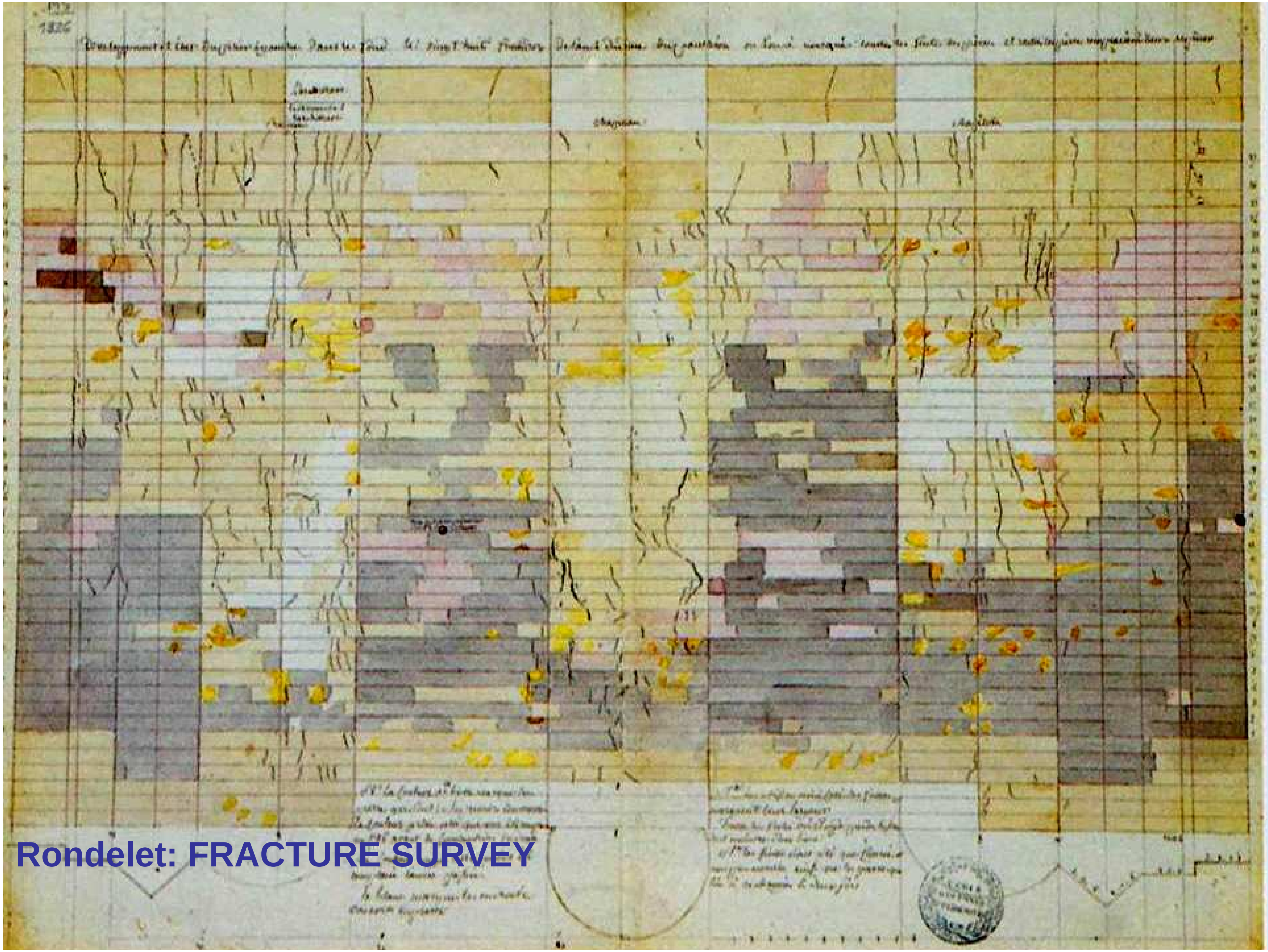
Average stress ≅ 1/10 strength by testing



SURVEY OF THE CRACKS IN THE PYLONS, DEFINITION OF THE CAUSES, STUDIES ABOUT THE THRUST IN THE DOMES AND DESIGN OF THE CONSOLIDATION WORKS



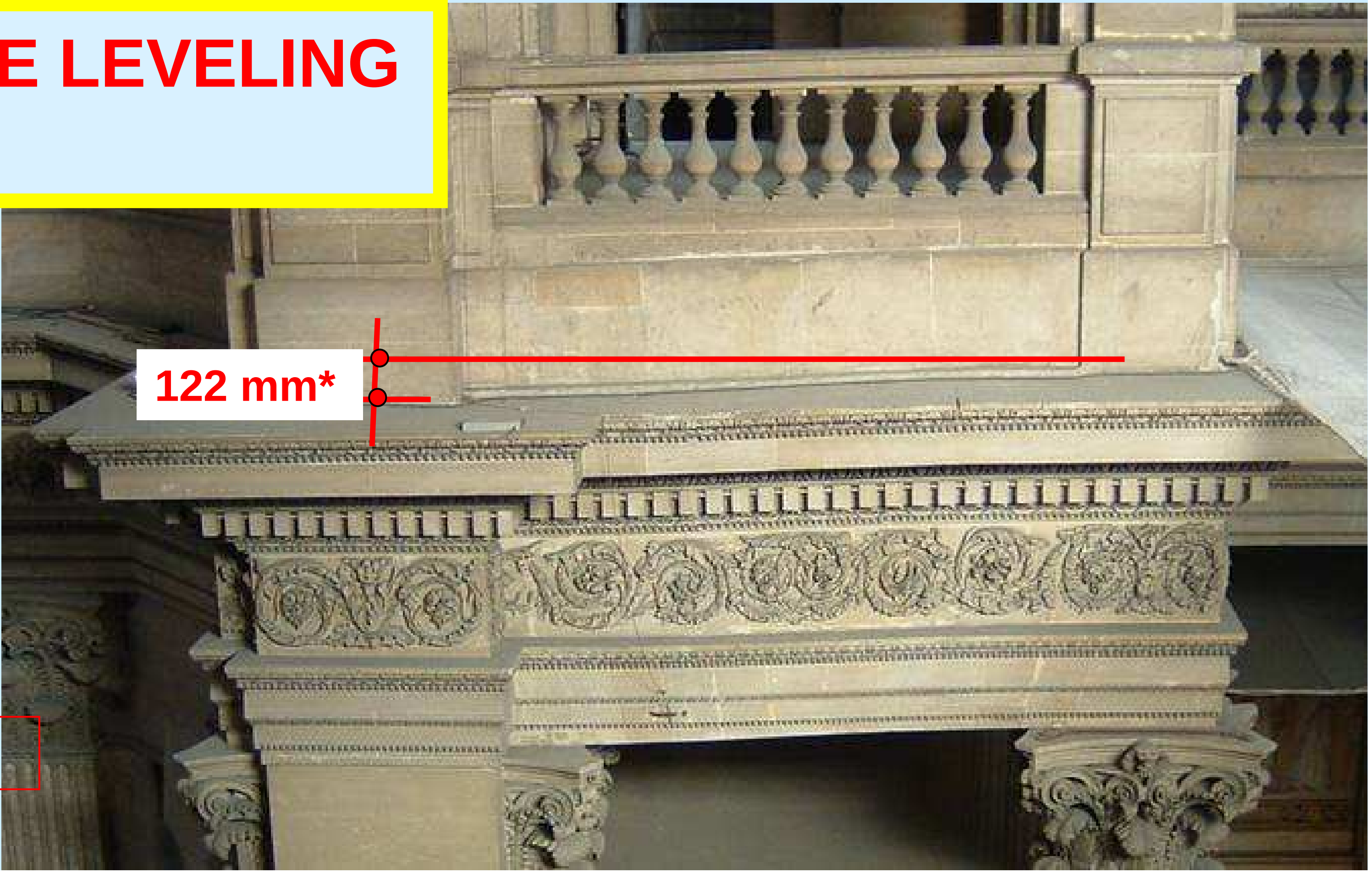
SECOND PHASE OF THE DEBATE : when the main cracks in the pylons of the dome first start to form (1780 - 1800)



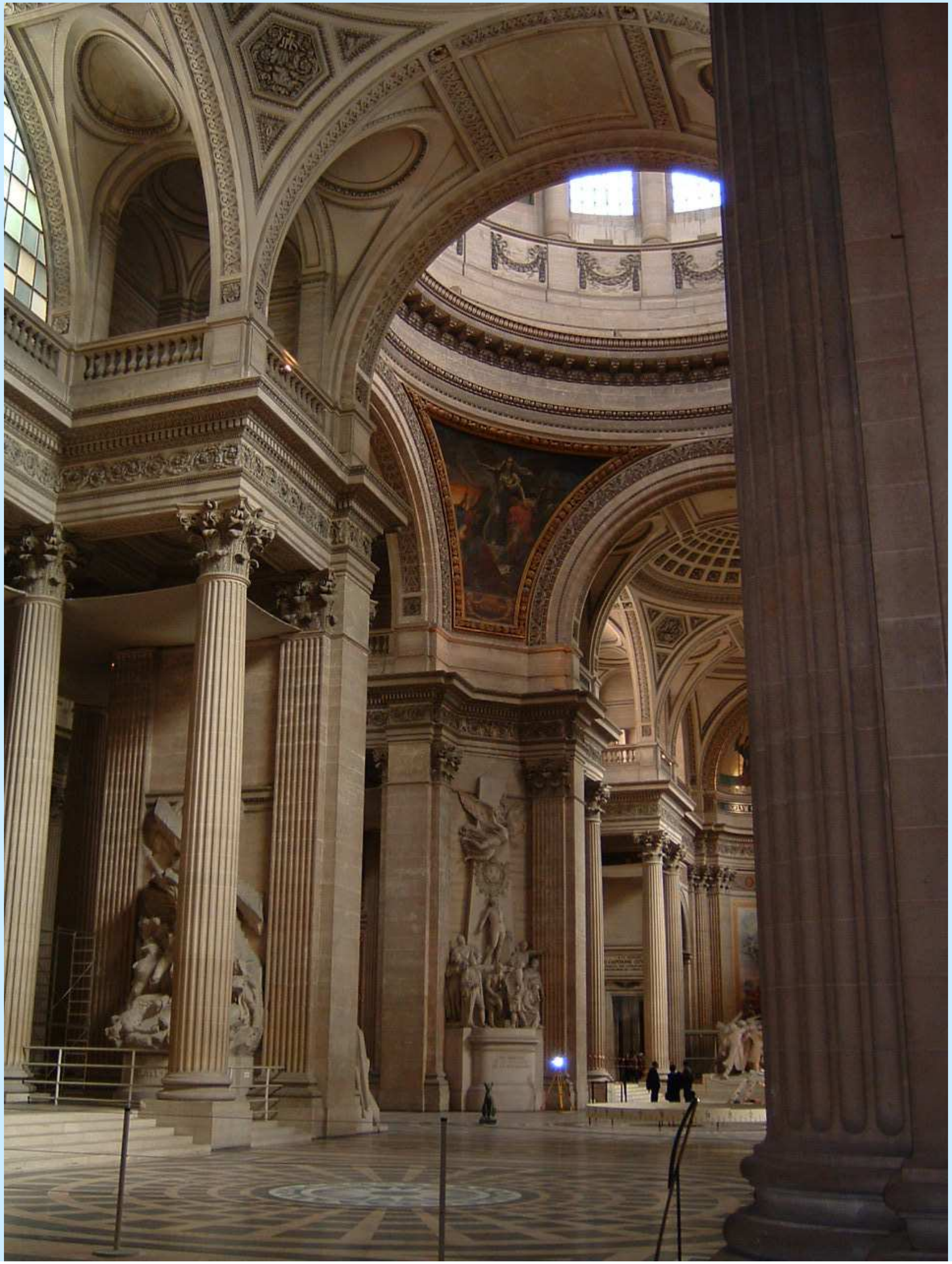
Rondelet: FRACTURE SURVEY

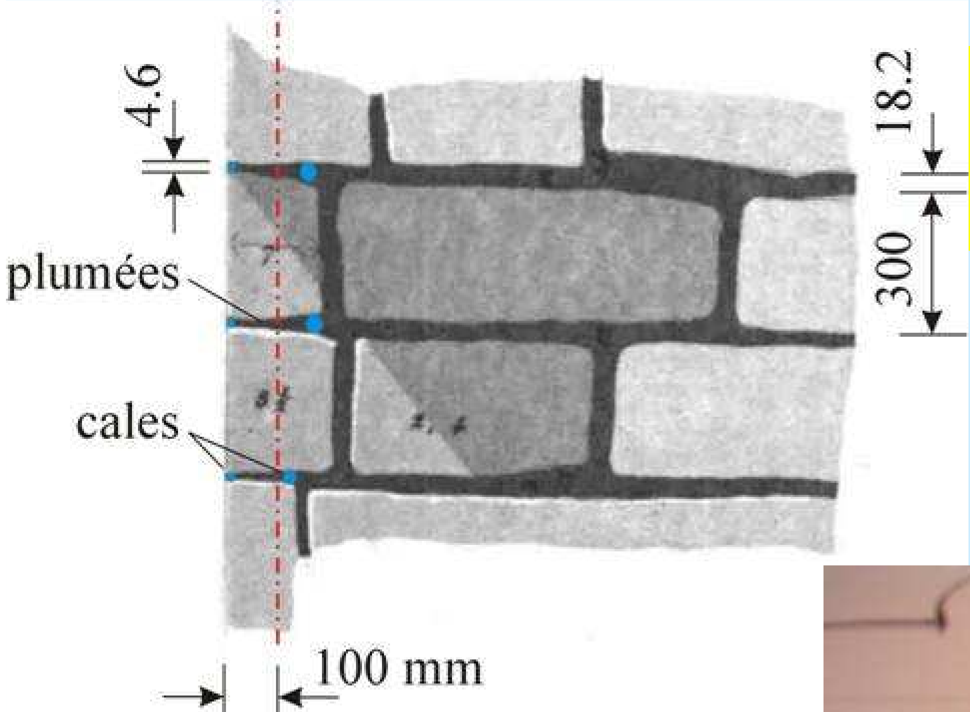
“Questo pilastro è quello che ha avuto il maggior cedimento. Grazie a dei controlli di livello effettuati in tempi diversi, si è potuto verificare che si è accorciato di 5 pollici e 2 linee e ½ (140 mm); dato che la *colonna isolata, che sostiene l’angolo della tribuna adiacente a sinistra del pilone ha avuto un accorciamento di sole 8 linee e ½ (18 mm), ne deriva una inclinazione di 4 pollici e ½ (122 mm) dell’architrave....* ”Rondelet

ACCURATE LEVELING (1796)



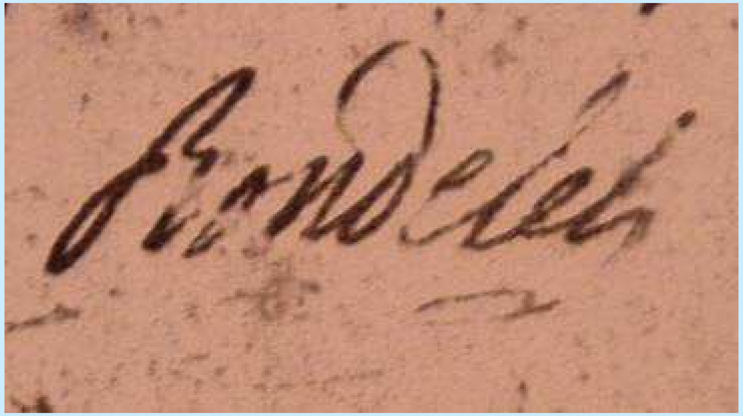
*Today 210 mm

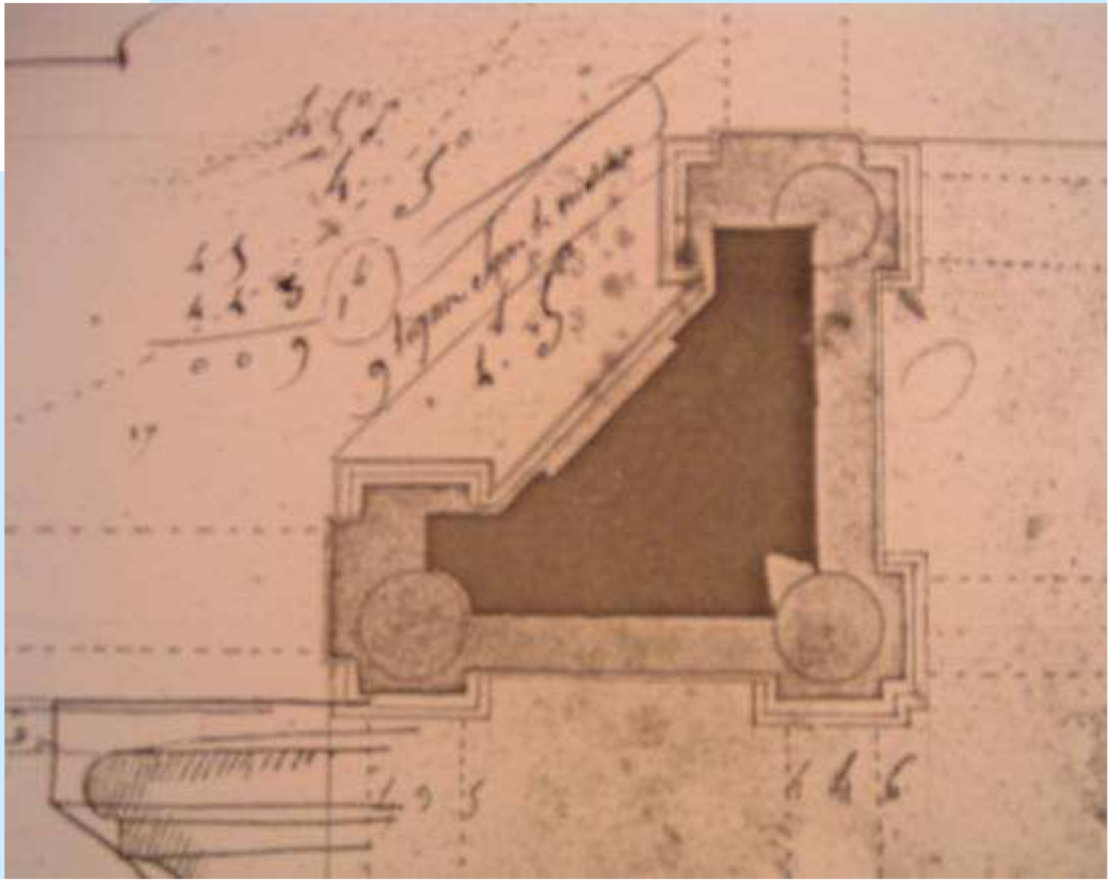




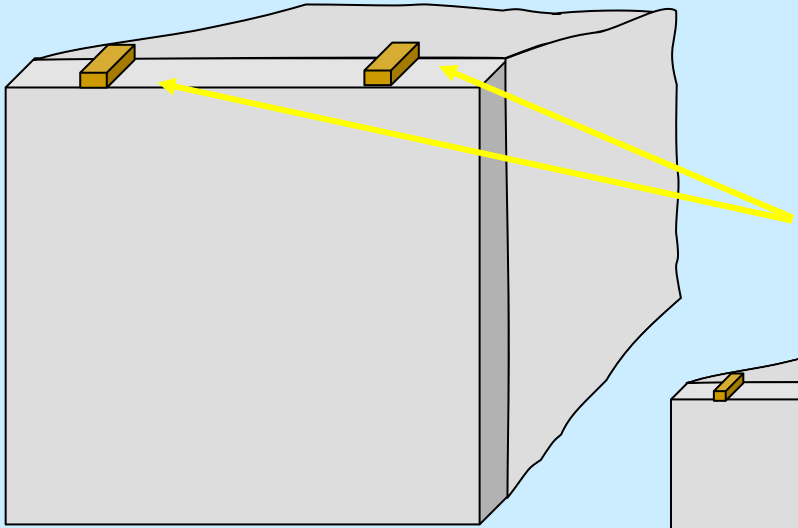
RONDELET'S EXPLANATION

TESTS

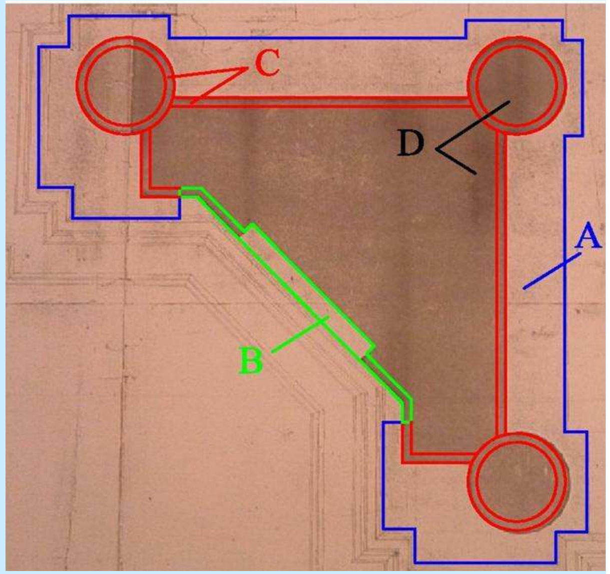


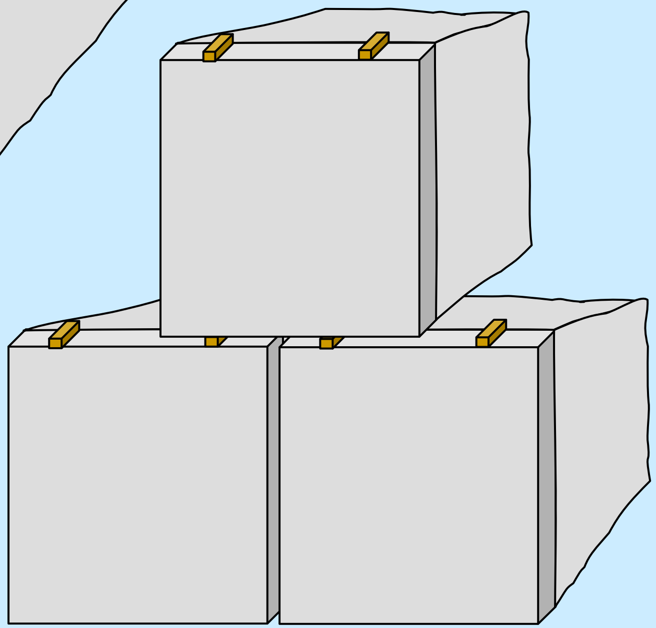


Also stress concentration due to oak wood wedges (cales)



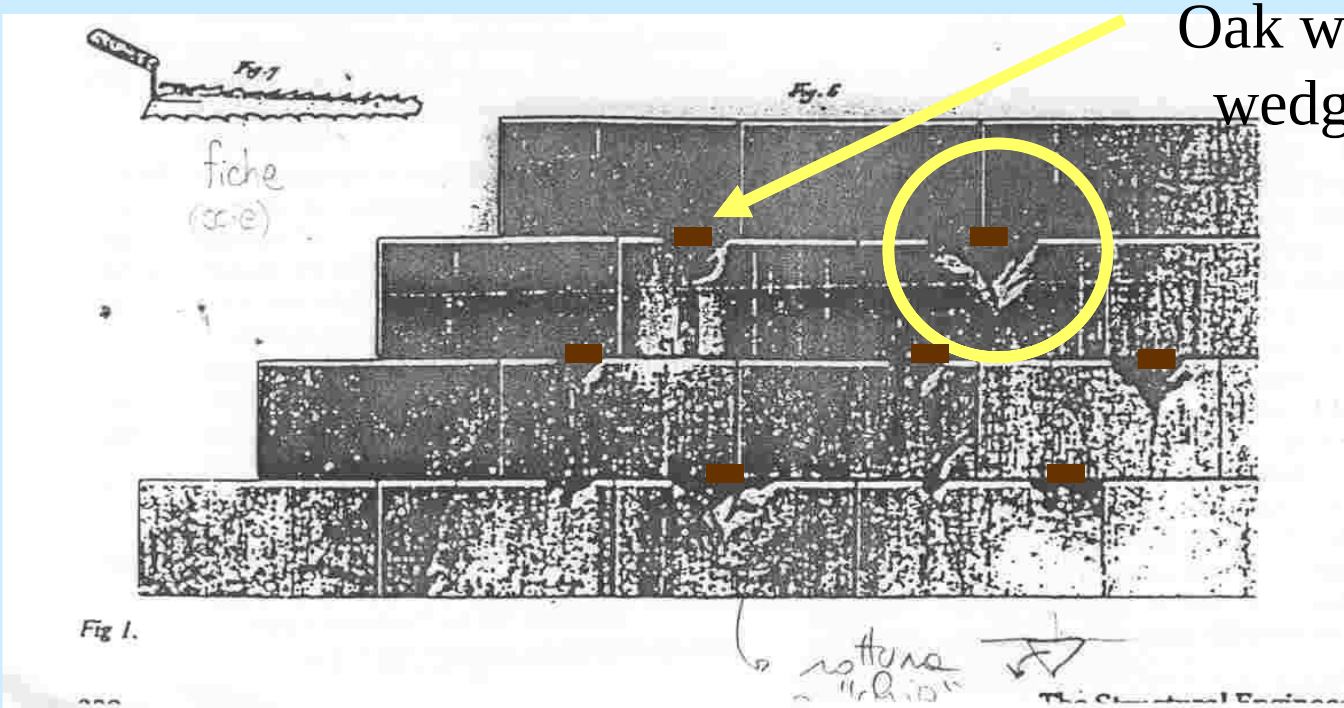
Oak wood wedges



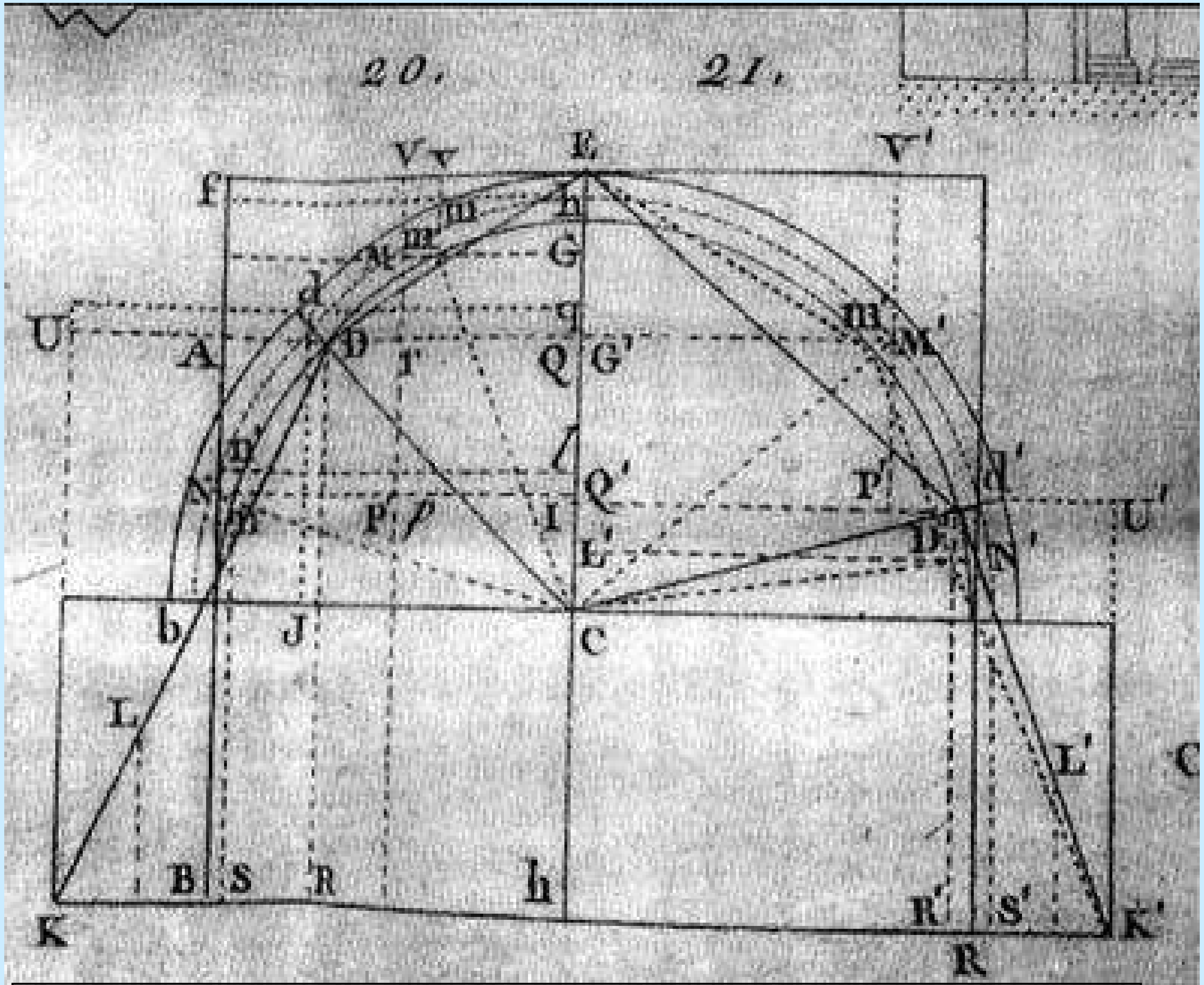


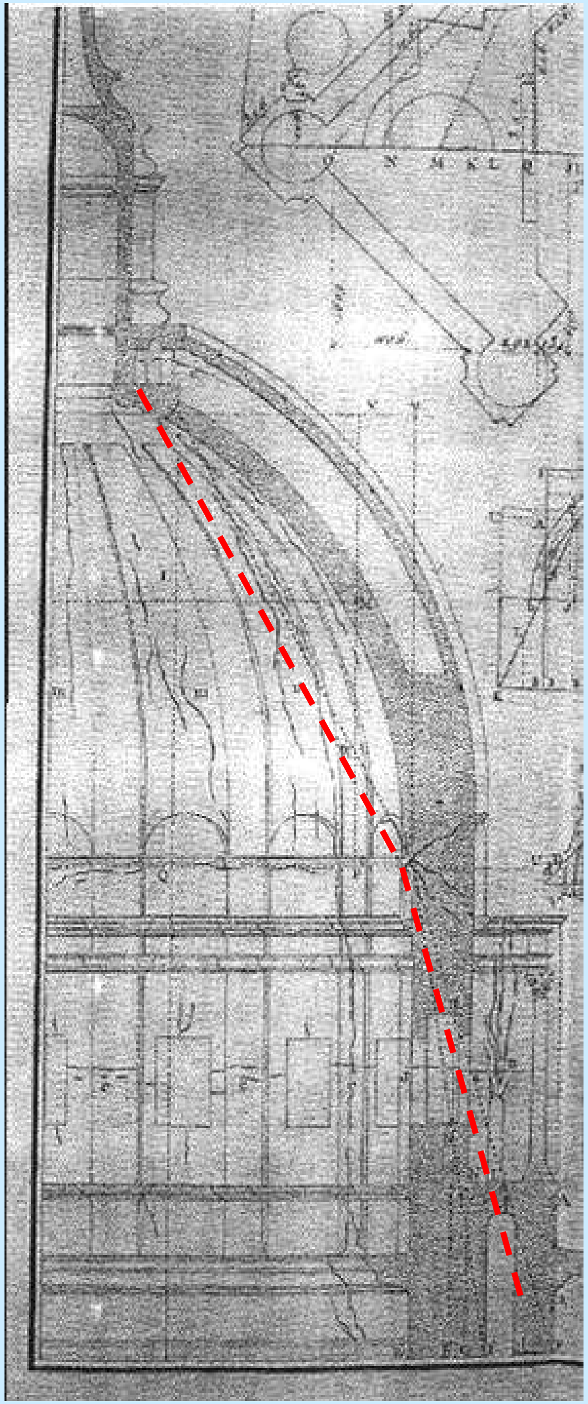
Assembly

Rondelet memory: cracks in the ashlar masonry of the pylon



Mortar shrinks as it dries and most of the load is taken through the slips of wood

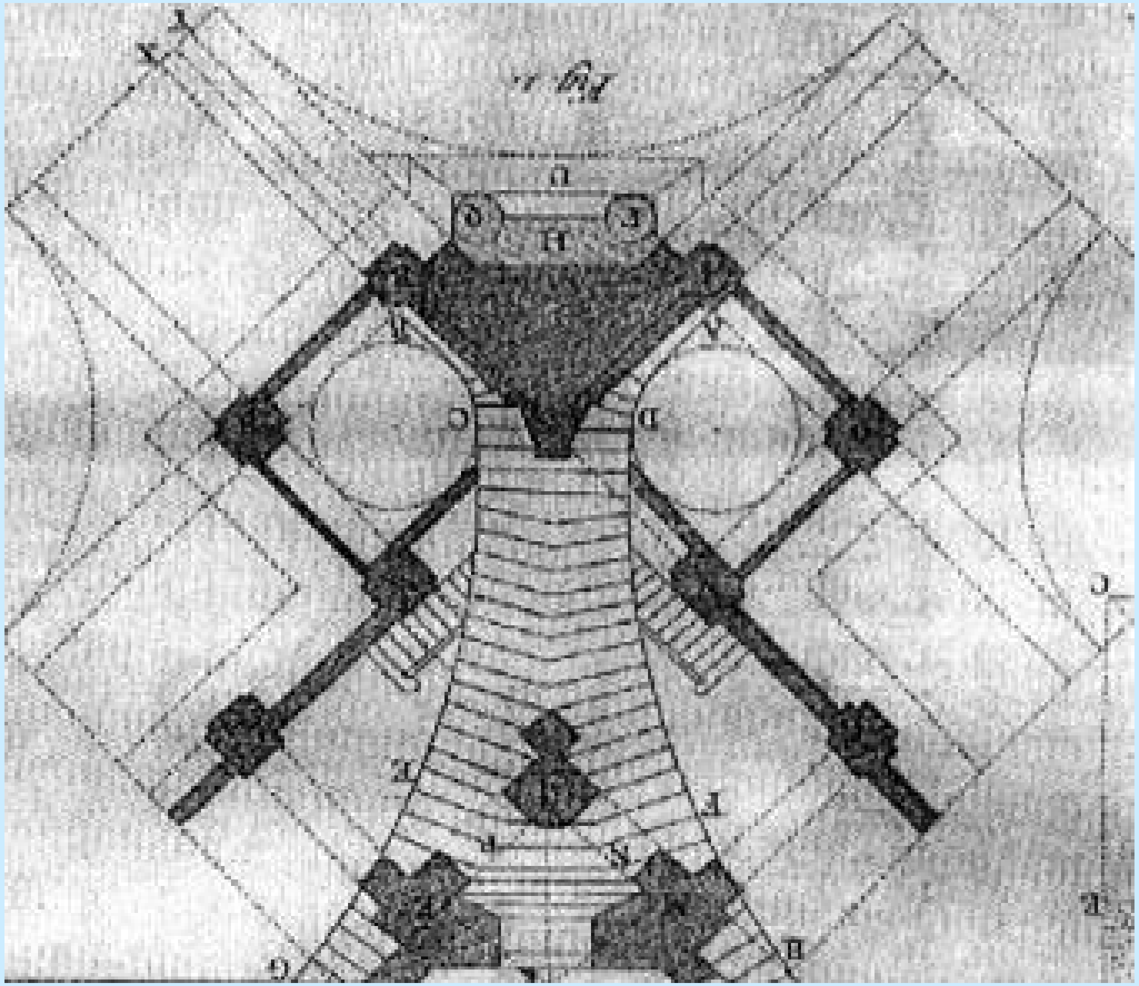


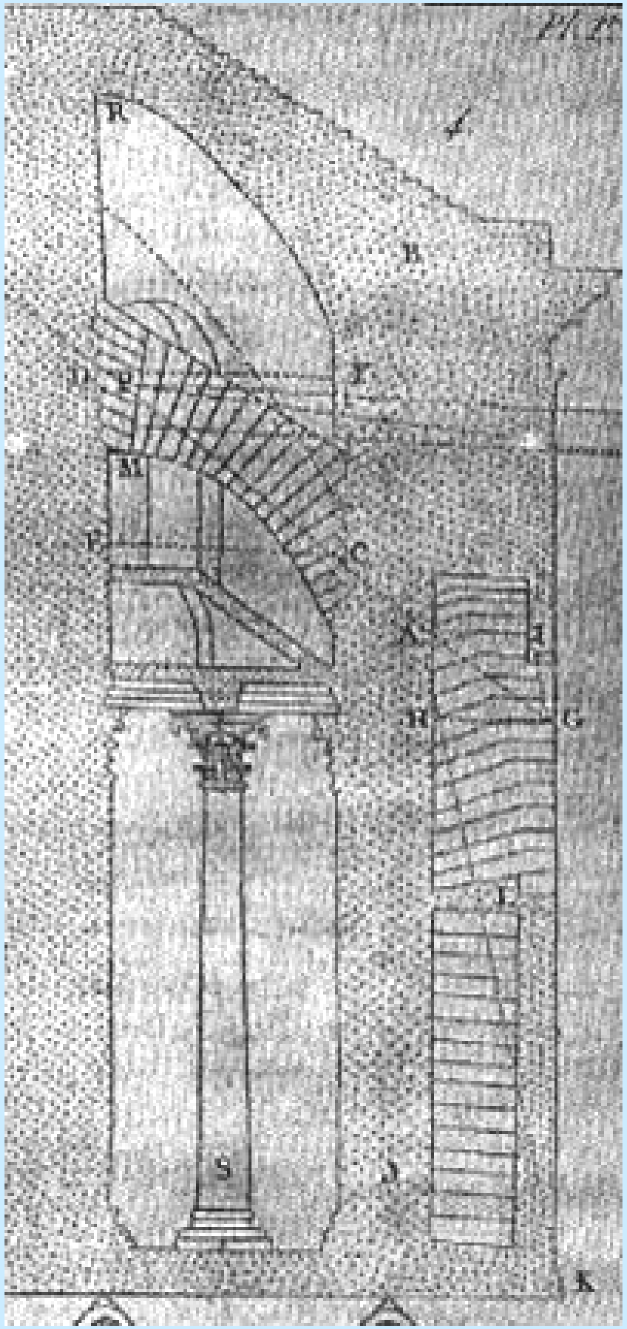


Gauthey (1796):

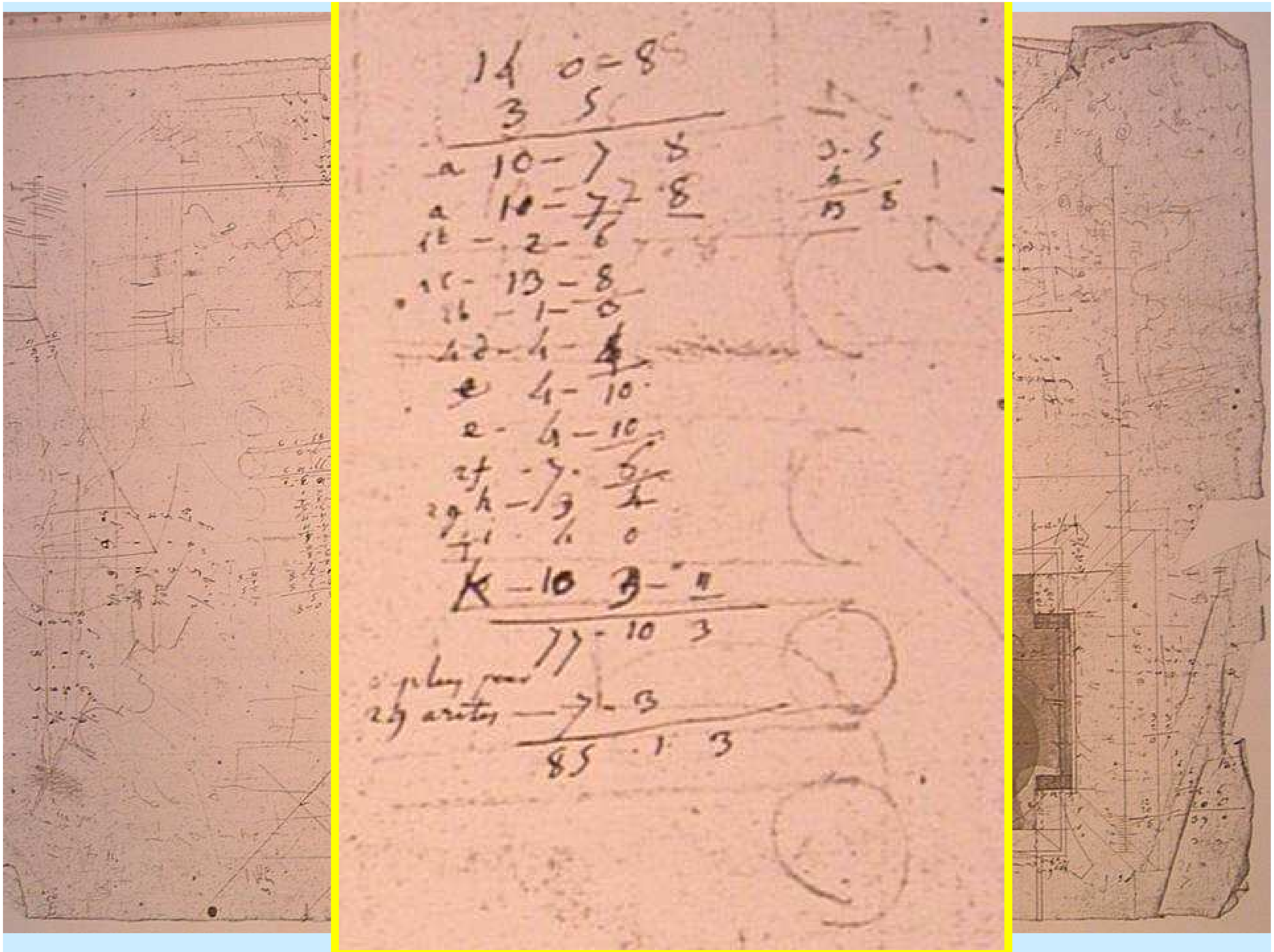
Calculates the thrust of the Pantheon dome.
Comparison with the dome of St. Peter in Rome

Thrust in domes?





Gauthey: Proposal for pillar strengthening


14 0 - 8
3 5
77 - 10 3
85 . 1 3


2005 02 24


30.08.2004


2005 02 24

Stress:

A: 1,37 Mpa

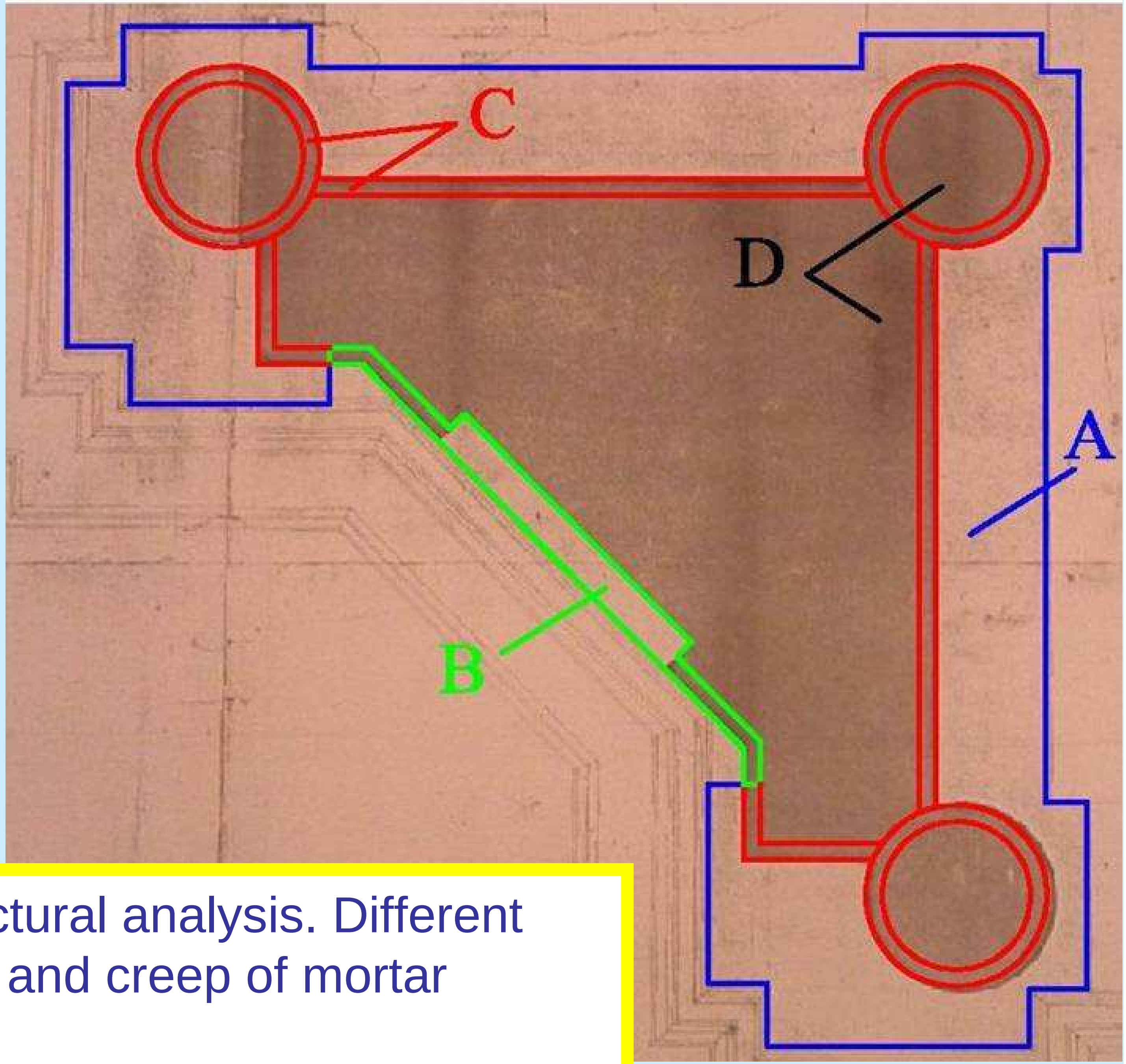
B: 0,50 Mpa

C: 9,00 Mpa

D: 1,18 Mpa

sagging:

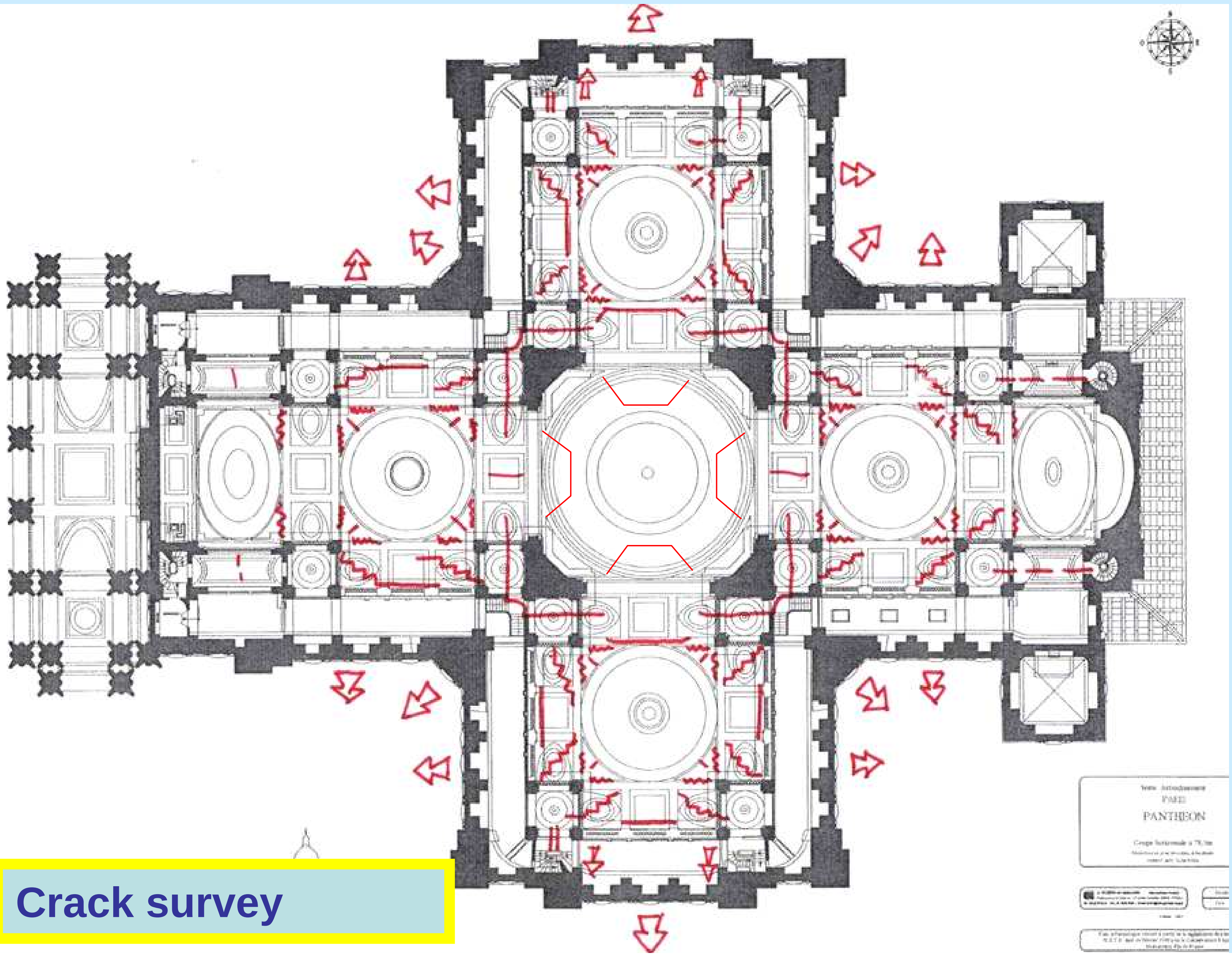
20-25 cm



Accurate structural analysis. Different elastic moduli and creep of mortar

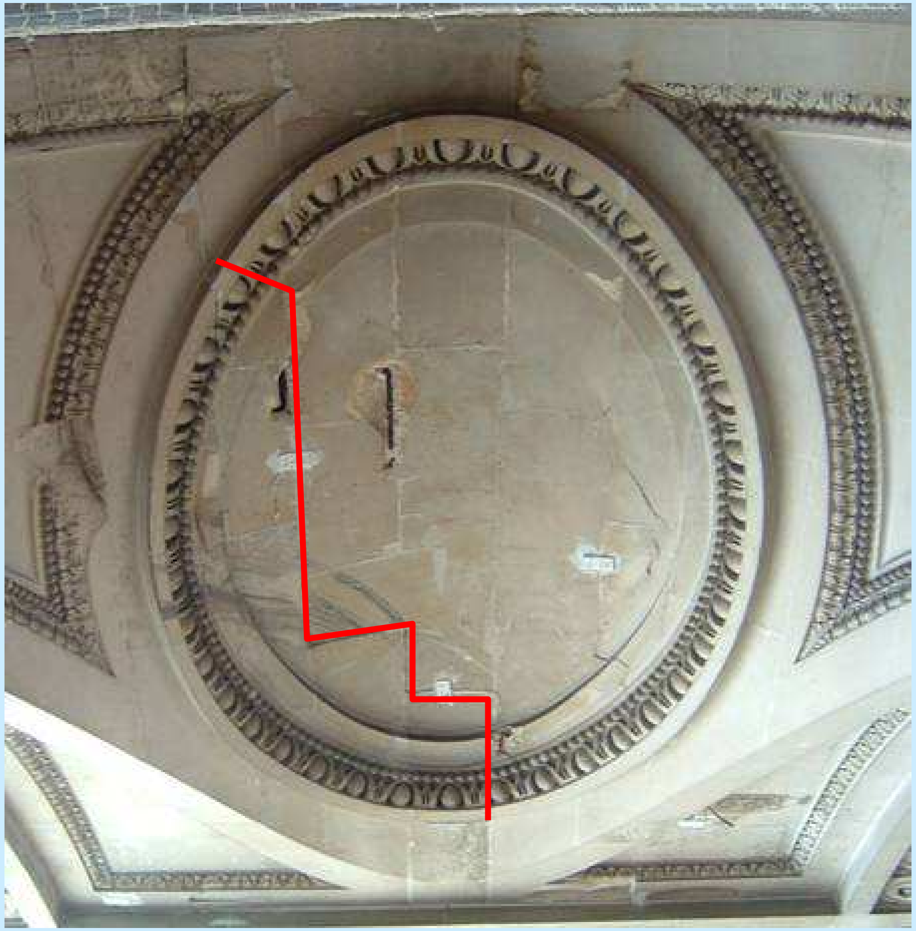
(D. Ferretti)

Third phase of the debate : Crack opening due to long-term phenomena; iron reinforcement oxidation (1980-2005)

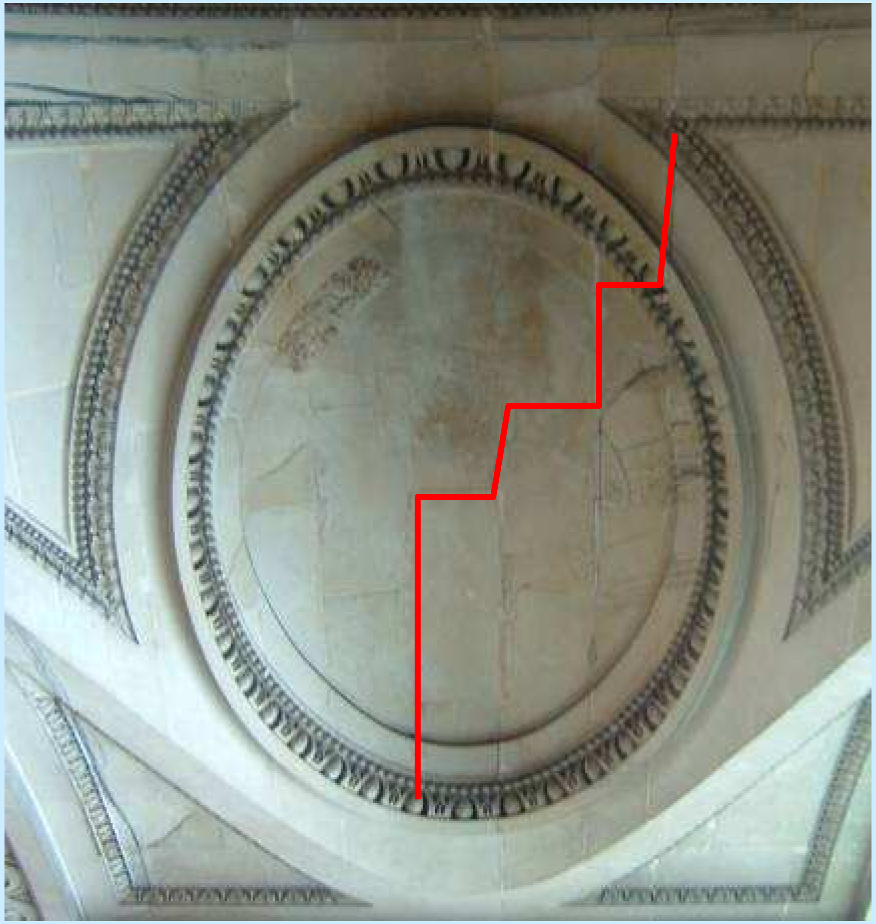


Crack survey

Symmetric crack paths in the lower circle



South-east nave



North-east nave


10-1-72
2005 02 24

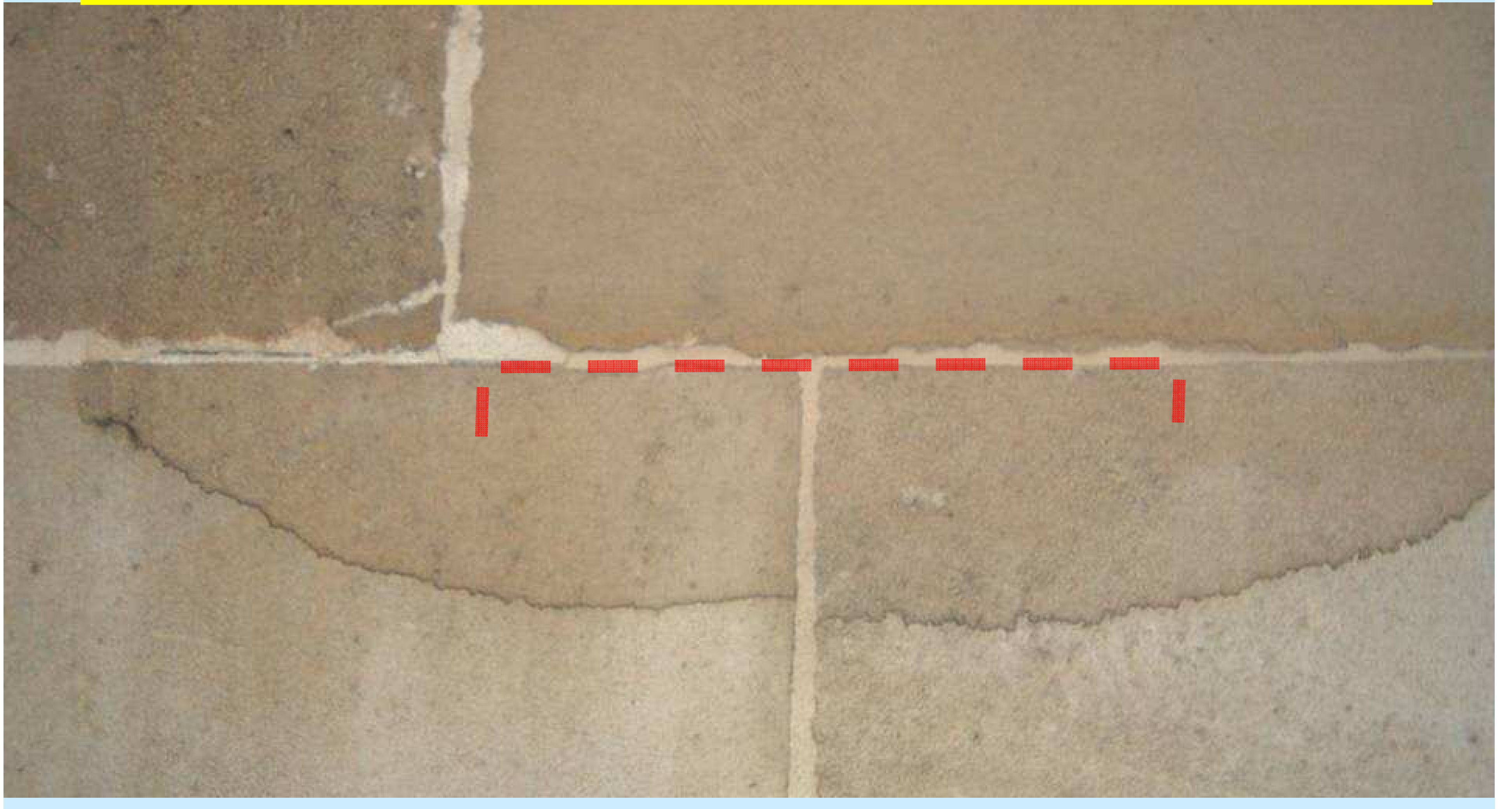

2005 02 24


2005 02 24


2005 02 24


30.08.2004

Typical crack path in the ashlar masonry

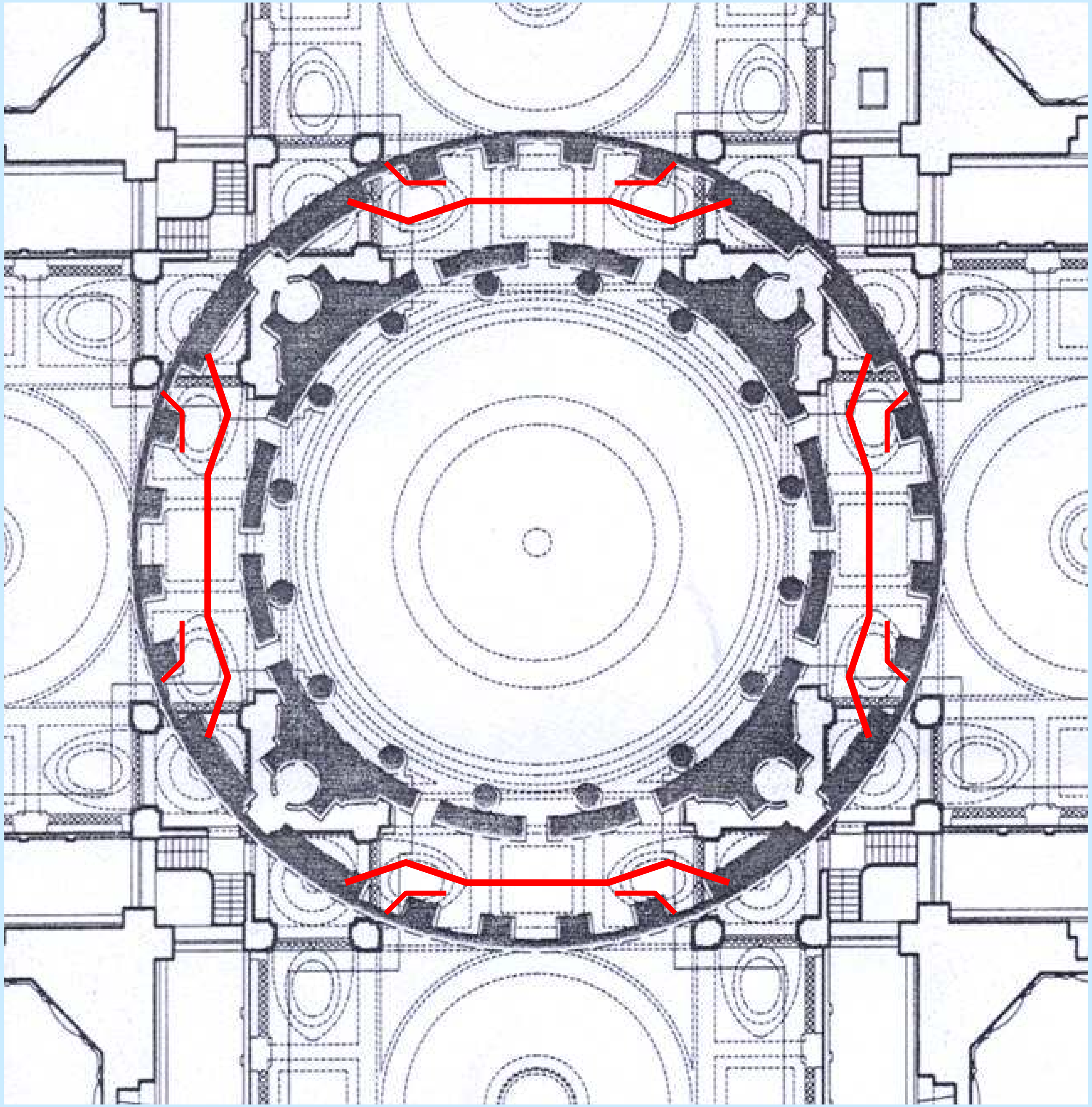


Damage in the ashlars

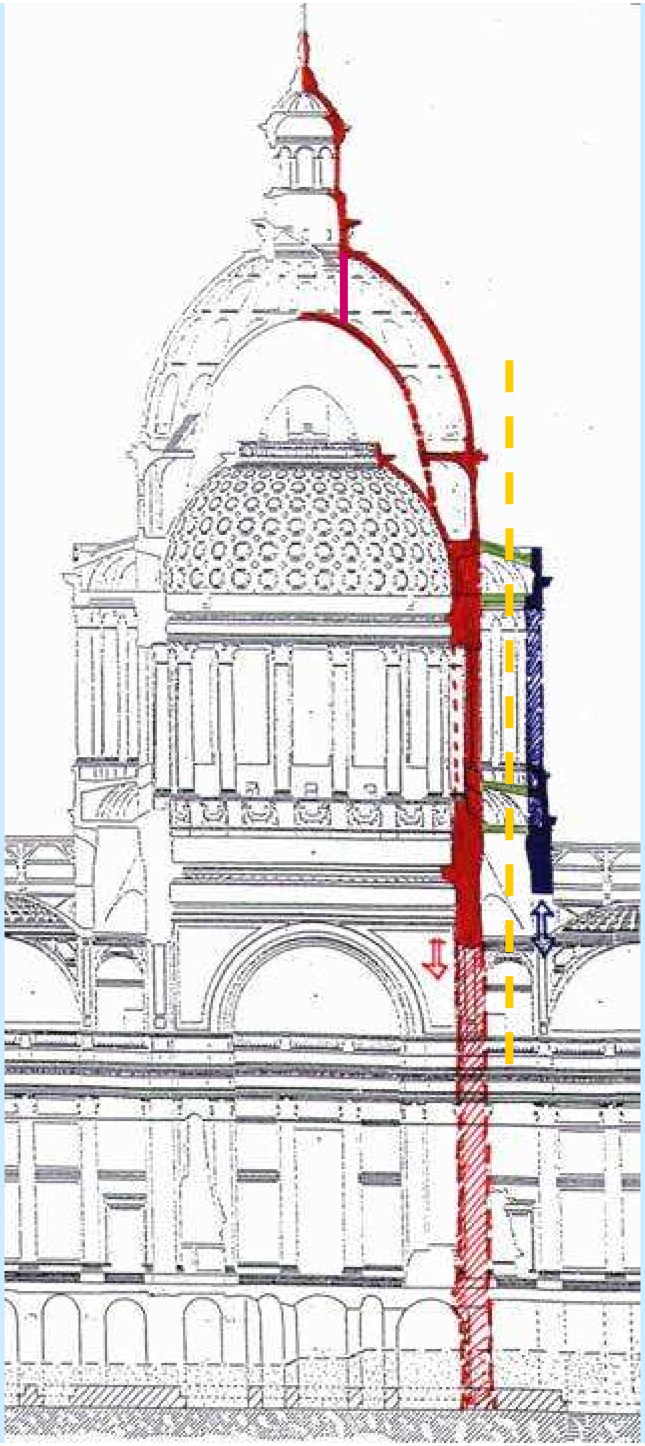
Is it provoked by the reinforcement expansion due to iron oxidation…

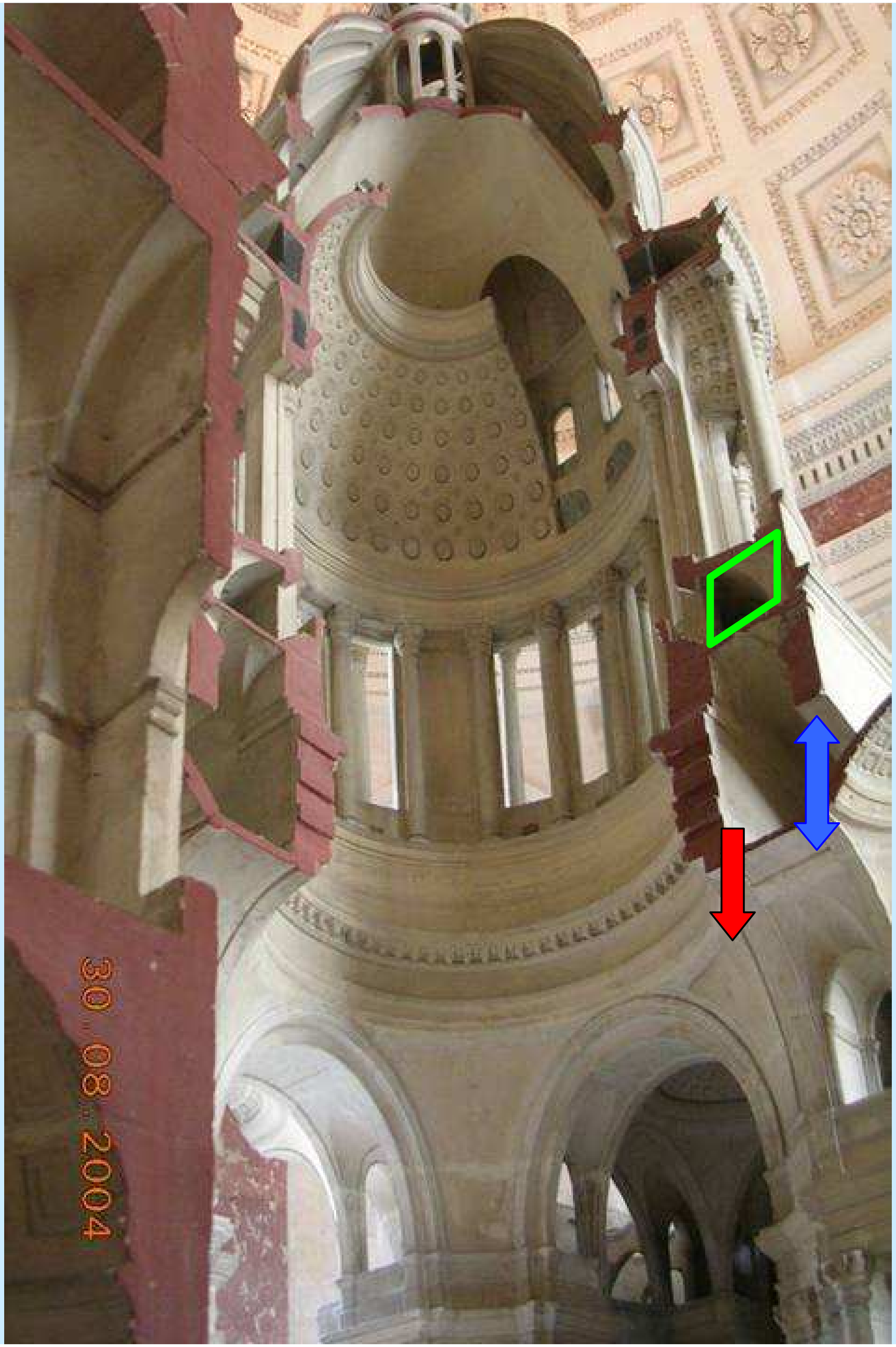
… or by the pull out of the staples

?



Galerie inférieure

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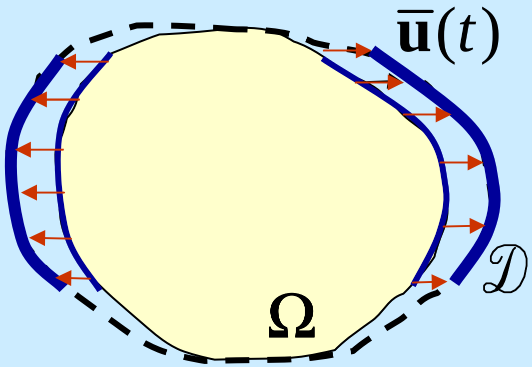
THE MODEL

Bourdin, Francfort, Marigo model (JMPS, 2000)



$$\mathbf{E} = sym(\nabla \mathbf{u}), \quad \mathrm{C} = 2\mu \mathrm{I} + \lambda \mathbf{I} \otimes \mathbf{I}, \quad k_\varepsilon \ll 1$$

$$[\varepsilon] = [L], \quad 0 < \varepsilon << diam(\Omega)$$

$$\Pi_\varepsilon[\mathbf{u}, s] = \int_\Omega \left[(s^2 + k_\varepsilon) \frac{1}{2} \mathrm{C}\mathbf{E}(\mathbf{u}) \cdot \mathbf{E}(\mathbf{u}) \right] da + \gamma \int_\Omega \left[\frac{\varepsilon}{2} |\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon} \right] da$$

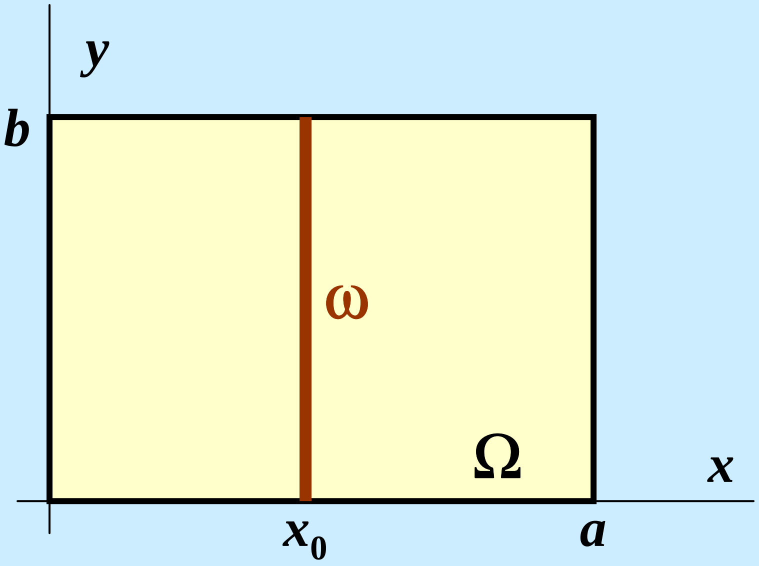
$$\min_{(u,s) \in \mathcal{A}_t} \Pi_\varepsilon[\mathbf{u}, s]$$

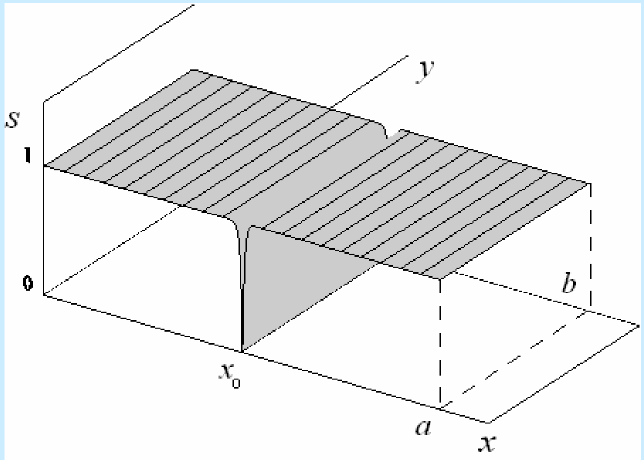
$$\mathcal{A}_t = \left\{ (\mathbf{u}, s) \in W^{1,2}(\Omega, \mathrm{R}^2) \times W^{1,2}(\Omega, \mathrm{R}) : \mathbf{u} = \bar{\mathbf{u}}(t) \text{ on } \mathcal{D} \text{ and } s = 1 \text{ on } \partial\Omega \right\}$$

$\dot{s} \le 0 \ \ \forall t, \quad s = 1$ at $t = 0$ **Irreversibility constraint**

Condition s = 1 on ∂Ω is different from BFM, JMPS 2000. No need of logic domain

Euristic argument





Suppose Ω is the rectangle and ω is straight and parallel to *y*

Assume that *s*(*x*,*y*) is of the type represented in the Figure with $s_{,y} = 0$:

We want to evaluate:

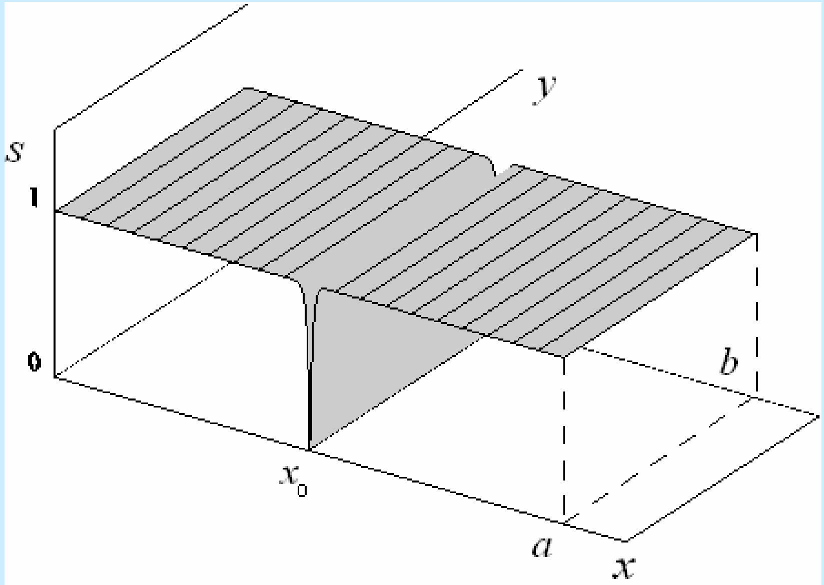
$$\gamma\int_{\Omega}\left[\frac{\varepsilon}{2}|\nabla s|^2+\frac{(1-s)^2}{2\varepsilon}\right]da$$

Euristic argument

$$\forall(\alpha,\beta)\in \mathrm{R}\times\mathrm{R}: \quad \alpha^2+\beta^2 \geq 2|\alpha||\beta| \text{ and } \alpha^2+\beta^2 = 2|\alpha||\beta| \; iff \; |\alpha|=|\beta|$$

$$\gamma\int_\Omega \left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon}\right] da \geq \gamma\int_\Omega \left[2\sqrt{\frac{\varepsilon}{2}}|\nabla s|\frac{(1-s)}{\sqrt{2\varepsilon}}\right] da$$

$$= \gamma\int_\Omega \left[|\nabla s|(1-s)\right] da$$



$$s,_y = 0$$

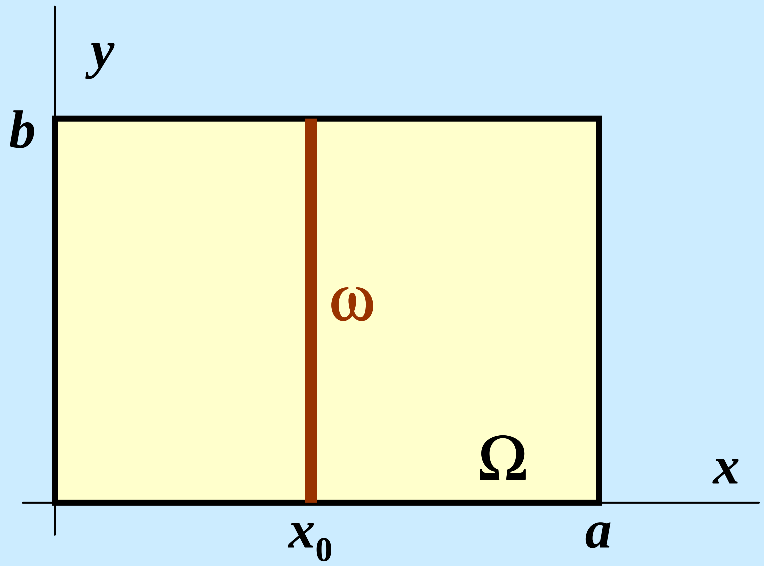
$$s,_x < 0 \text{ for } 0 < x < x_0 \; ,$$

$$s,_x > 0 \text{ for } x_0 < x < a \; .$$

Euristic argument

$$\gamma\int_{\Omega}\left[\left|\nabla s\right|(1-s)\right]da = \gamma\int_0^b\left(\int_0^{x_0} -s_{,x}(1-s)dx\right)dy + \gamma\int_0^b\left(\int_{x_0}^{a} s_{,x}(1-s)dx\right)dy$$

$$= \gamma\int_0^b\left(\int_0^{x_0} \partial_x[(1-s)^2/2]dx\right)dy + \gamma\int_0^b\left(\int_{x_0}^{a} -\partial_x[(1-s)^2/2]dx\right)dy = \gamma b = \gamma \text{ meas } \omega$$



γ plays the role of fracture energy

Euristic argument

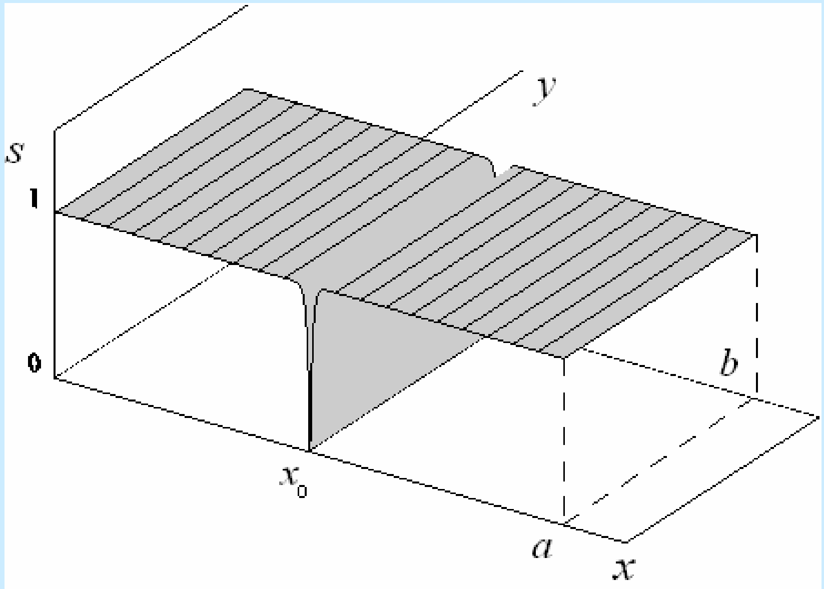
The optimal profile that attains the lower bound can be easily calculated

$$\gamma\int_\Omega\left[\frac{\varepsilon}{2}|\nabla s|^2+\frac{(1-s)^2}{2\varepsilon}\right]da=\gamma\int_\Omega\Big[|\nabla s|(1-s)\Big]da \text{ when } \sqrt{\frac{\varepsilon}{2}}|\nabla s|=\frac{|1-s|}{\sqrt{2\varepsilon}}$$

$$\forall(\alpha,\beta)\in \mathrm{R}\times\mathrm{R}: \quad \alpha^2+\beta^2\geq 2|\alpha||\beta| \text{ and } \alpha^2+\beta^2=2|\alpha||\beta| \text{ } iff \text{ } |\alpha|=|\beta|$$

From this condition find:

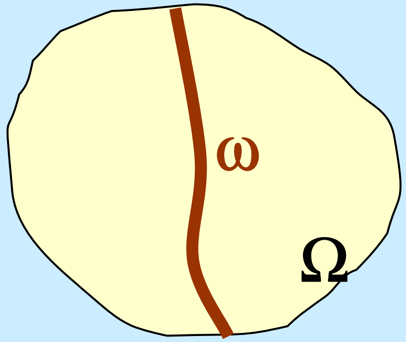
$$s(x)=1-e^{-\frac{|x-x_0|}{\varepsilon}}$$



Γ-convergence result

$$\Pi_\varepsilon[\mathbf{u}, s] = \int_\Omega \left[(s^2 + k_\varepsilon)\frac{1}{2}\mathrm{C}\mathbf{E}(\mathbf{u})\cdot\mathbf{E}(\mathbf{u}) \right] da + \gamma \int_\Omega \left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon} \right] da$$

$$\Gamma \lim_{\varepsilon \to 0} \; (\Pi_\varepsilon[\mathbf{u}, s]) = \Pi[\mathbf{u}, \omega]$$



ω is the crack location

$$\Pi[\mathbf{u}, \omega] = \int_{\Omega\backslash\omega} \frac{1}{2}\mathrm{C}\mathbf{E}(\mathbf{u})\cdot\mathbf{E}(\mathbf{u})\, da + \gamma\, meas(\omega)$$

Griffith crack model

Bourdin, Francfort, Marigo model REVISITED

B. F. M. model:

$$\Pi_\varepsilon[\mathbf{u}, s] = \int_\Omega \left[(s^2 + k_\varepsilon)\frac{1}{2}\mathrm{C}\mathbf{E}(\mathbf{u})\cdot\mathbf{E}(\mathbf{u}) \right] da + \gamma \int_\Omega \left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon} \right] da$$

Proposal:

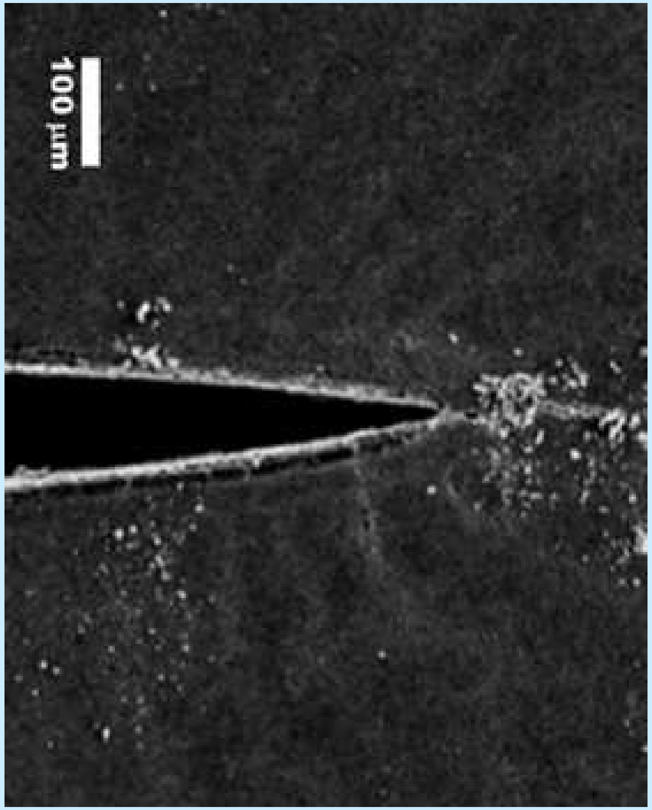
$$\Pi_\varepsilon^D[\mathbf{u}, s] = \int_\Omega \left[(s^2 + k_\varepsilon)\frac{1}{2}\mathrm{C}\mathbf{E}_{dev}(\mathbf{u})\cdot\mathbf{E}_{dev}(\mathbf{u}) \right] da + \int_\Omega \left[(1 + k_\varepsilon)\frac{1}{2}\mathrm{C}\mathbf{E}_{sph}(\mathbf{u})\cdot\mathbf{E}_{sph}(\mathbf{u}) \right] da$$
$$+ \gamma \int_\Omega \left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon} \right] da,$$

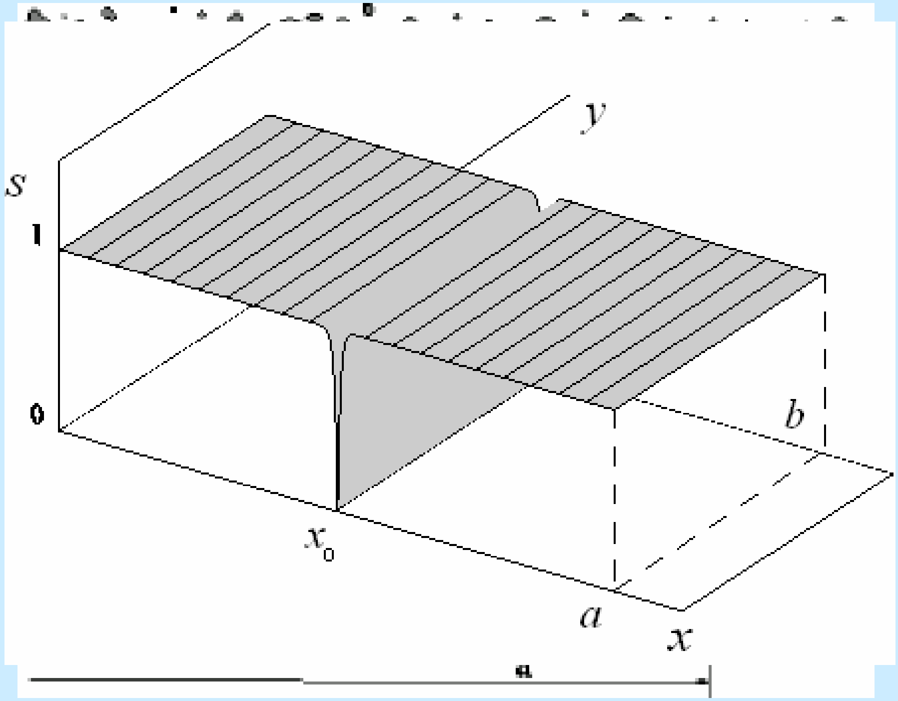
$$\mathbf{E}_{dev}(\mathbf{u}) = \mathbf{E}(\mathbf{u}) - \frac{1}{3} tr\mathbf{E}(\mathbf{u})\,\mathbf{I}\ , \quad \mathbf{E}_{sph}(\mathbf{u}) = \frac{1}{3} tr\mathbf{E}(\mathbf{u})\,\mathbf{I}\ ,$$

Mode II fracture governed by von Mises criterion

Damage model and process zone

Process zone at crack tip





s **plays the role of a damage parameter**

s = 1 sound material; s = 0 damaged material

Material Parameters

$$\Pi_\varepsilon^D[\mathbf{u}, s] = \int_\Omega \Big[(s^2 + k_\varepsilon) W_{dev}(\mathbf{u}) \Big]\, da + \int_\Omega \Big[(1 + k_\varepsilon)\, W_{sph}(\mathbf{u}) \Big]\, da + \gamma \int_\Omega \left[\frac{\varepsilon}{2} |\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon} \right] dx,$$

Boundary Condition s=1 on ∂Ω is acceptable because of gluing

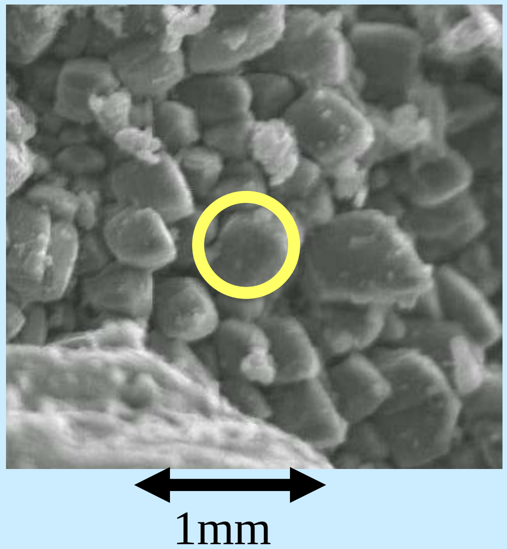
Calcareous rock

$$\mu = 4545\ Mpa\ ;\ \ \lambda = 1136\ Mpa\ \ (E = 10000\ Mpa\ ;\ \ \nu = 0.1)$$

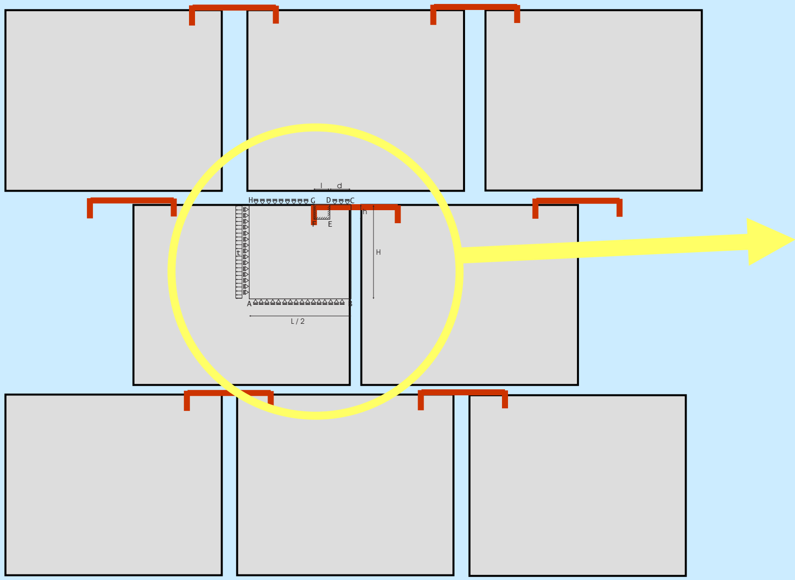
$$\gamma = 25 \cdot 10^{-3}\ N/mm\ \ ,\ \ \ K_\varepsilon = 10^{-2}$$

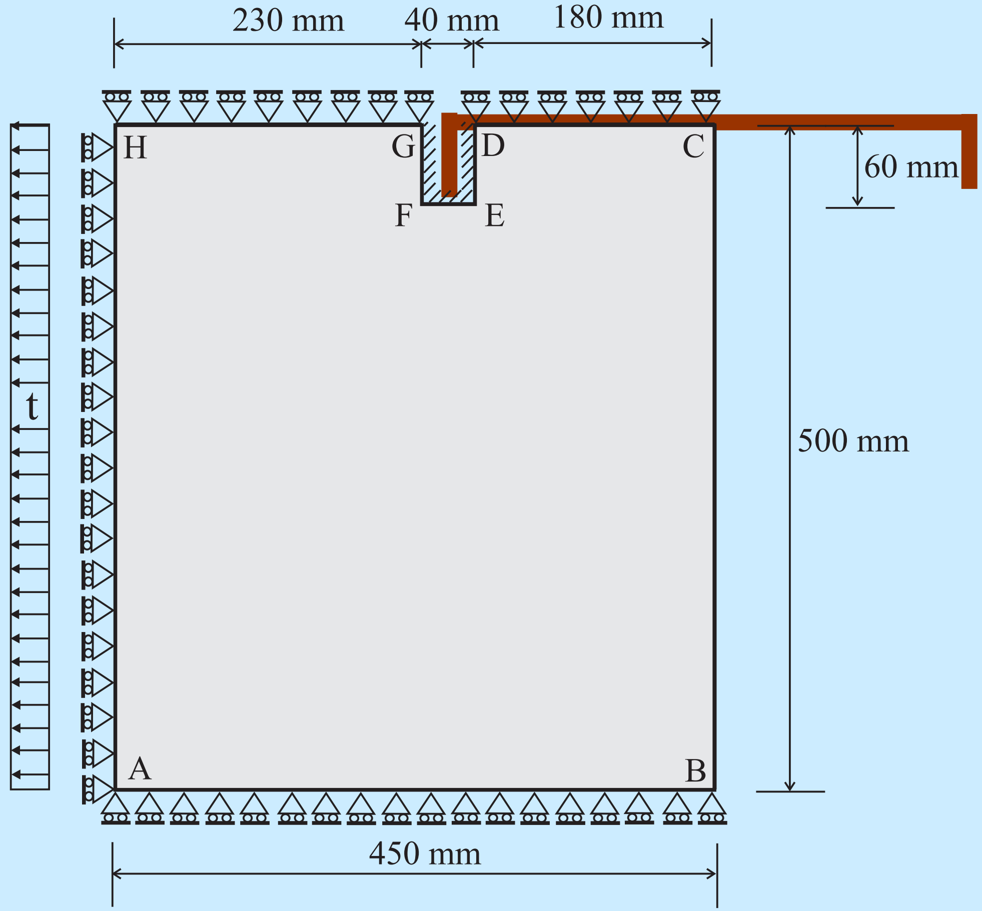
ε represents the internal characteristic length ≅ 2 ÷ 3 D

D ≅ 0.5÷ 1 mm ⇒ ε = 2 mm



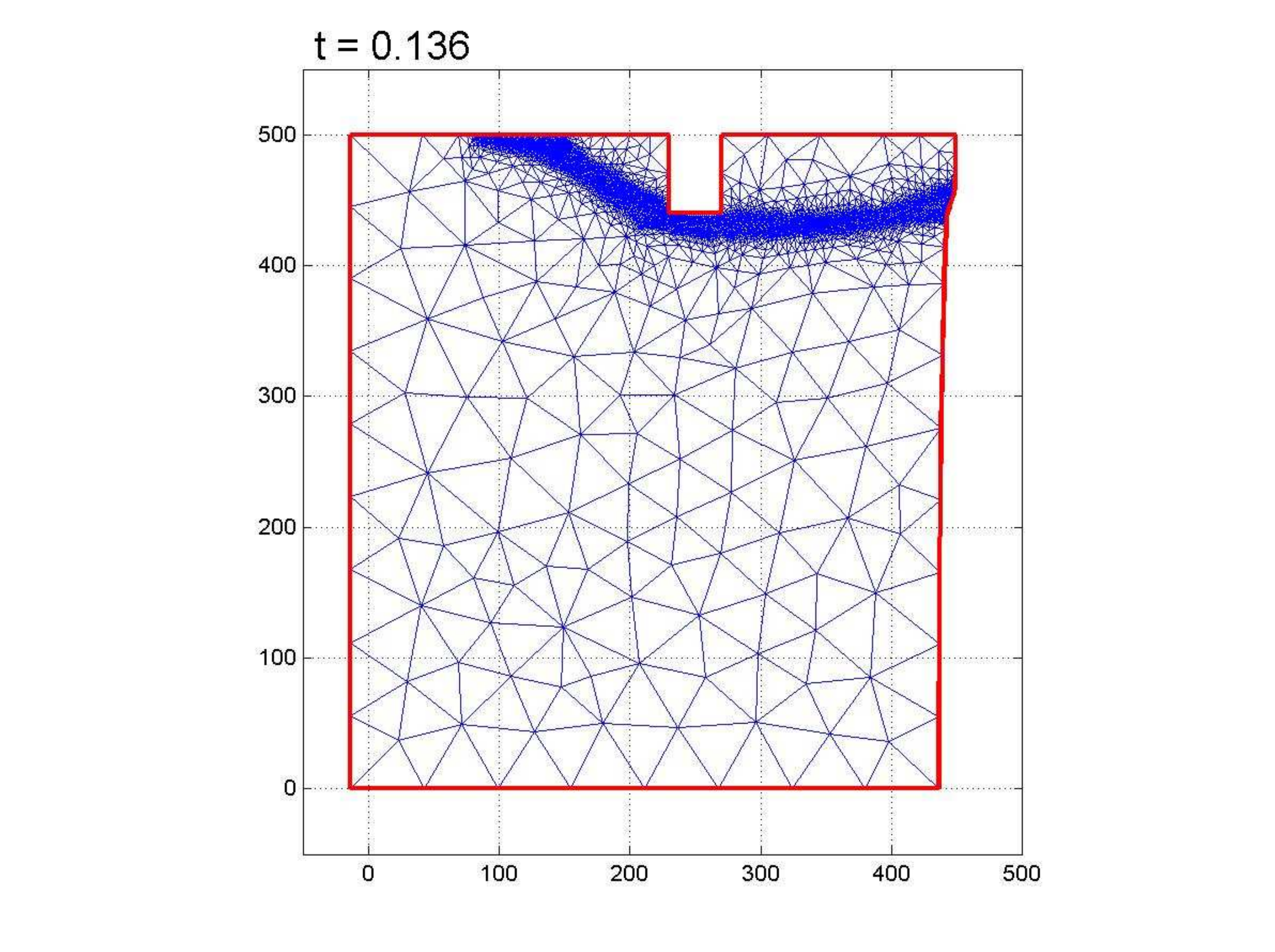
Numerical experiment. Cramp pull





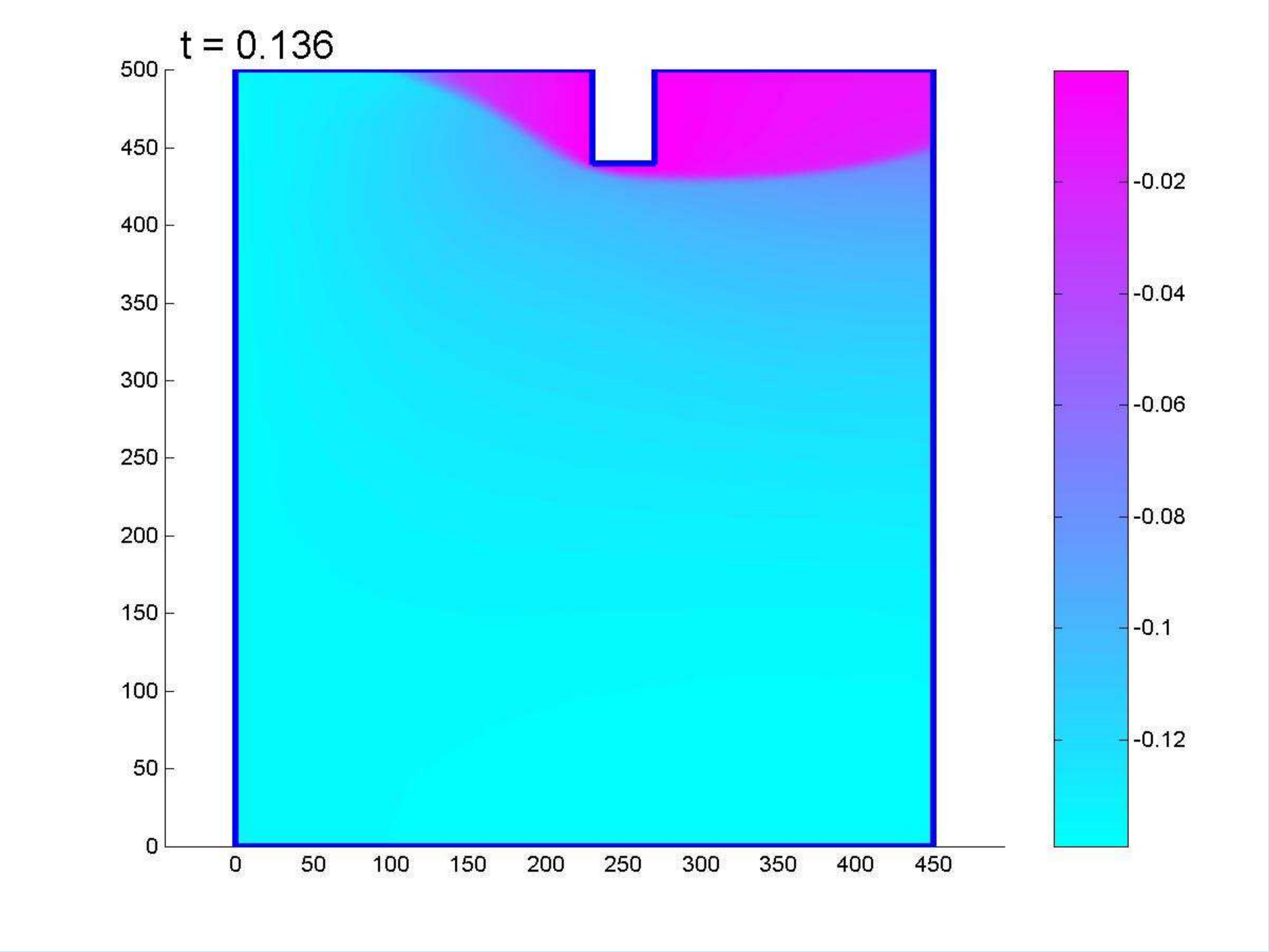
Cramp Pull – Deformed Mesh

t = clamp pull (mm). Displacement amplification = 10^2

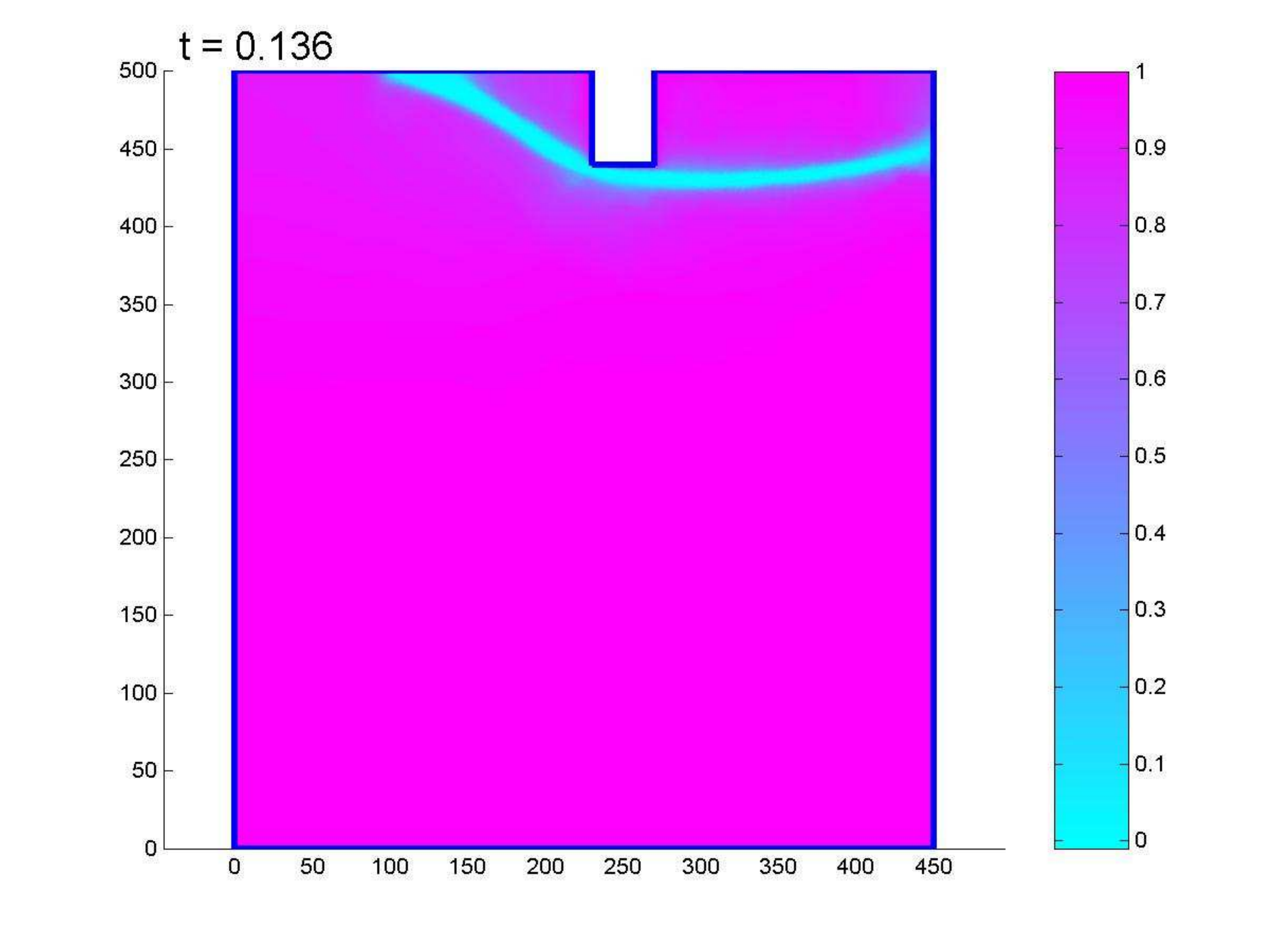


Cramp-pull - horizontal displacement

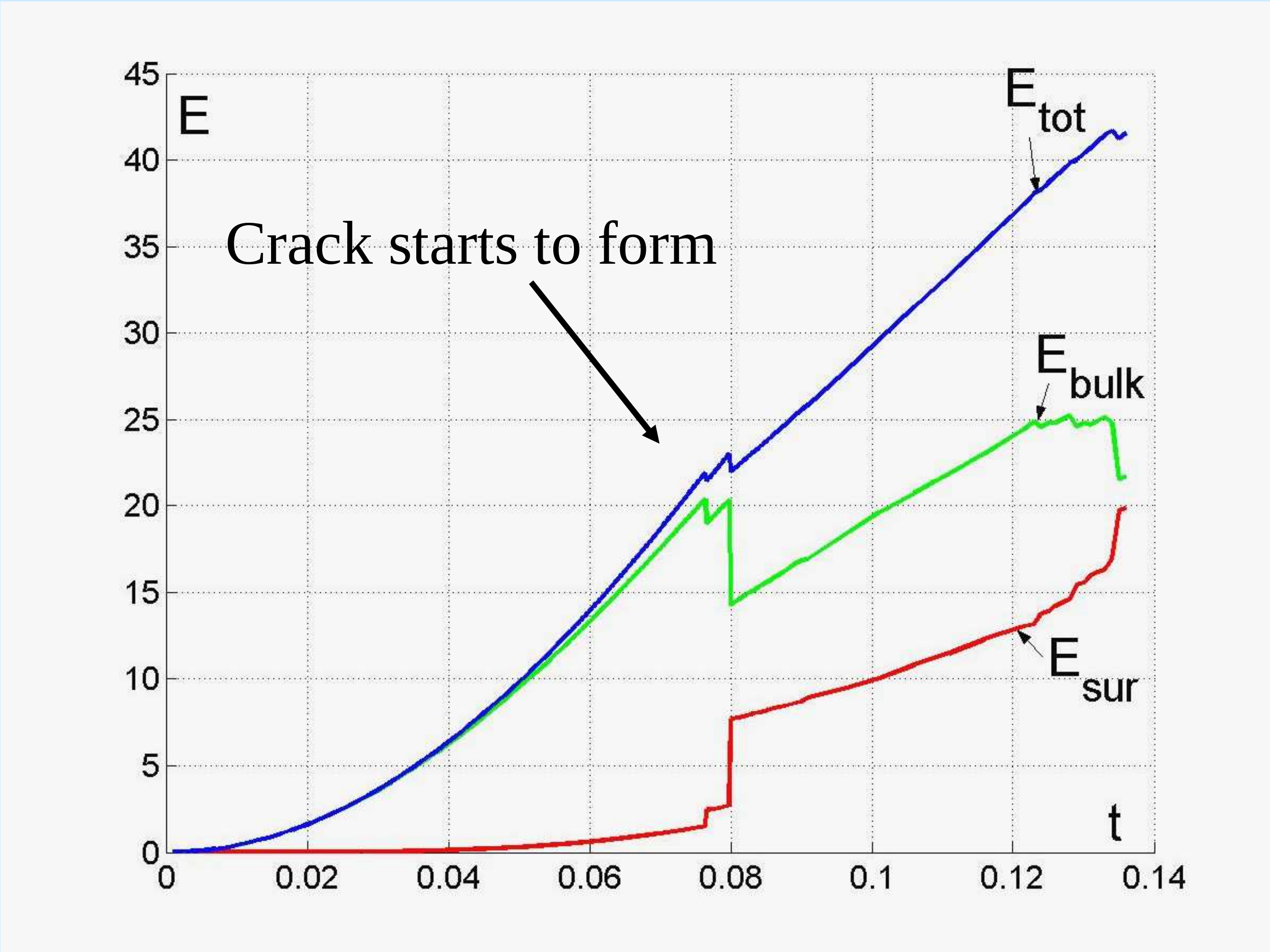
t = clamp pull (mm). Displacement (mm)



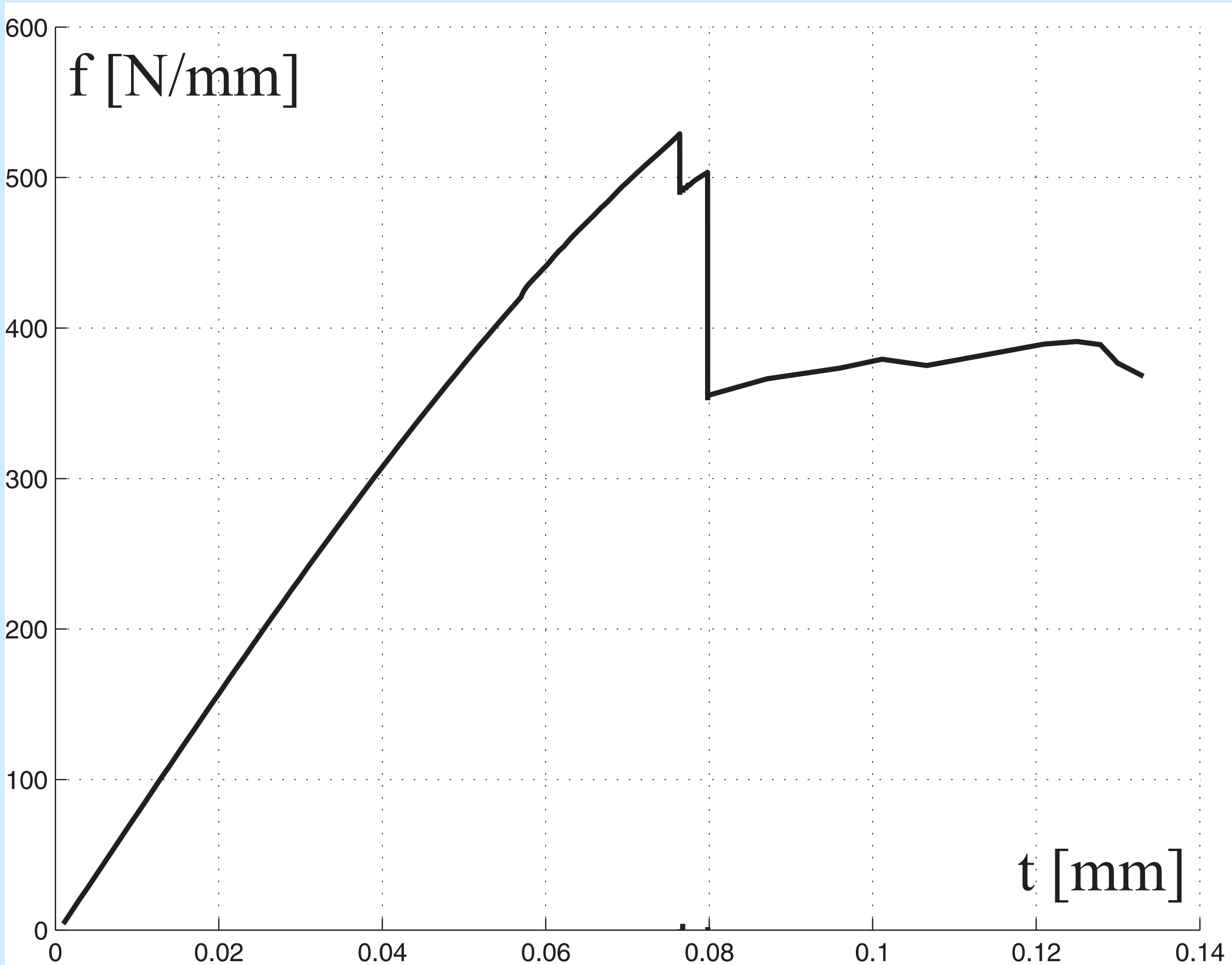
Cramp pull – damage field s



Cramp Pull – bulk and surface energy

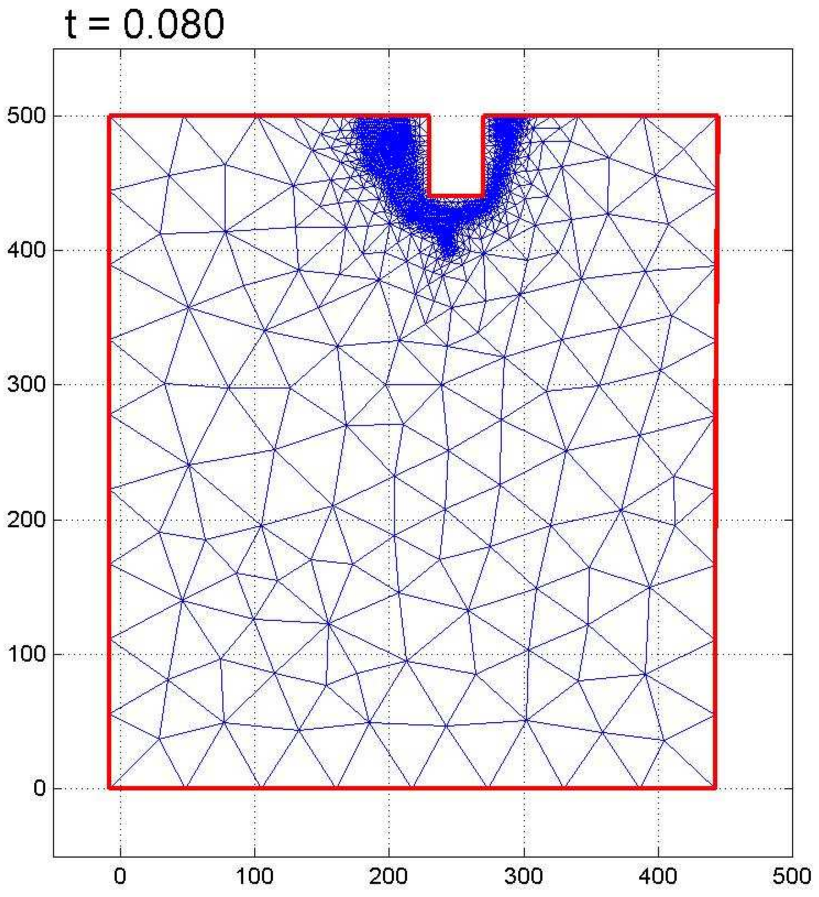


Cramp Pull – Force vs. displacement



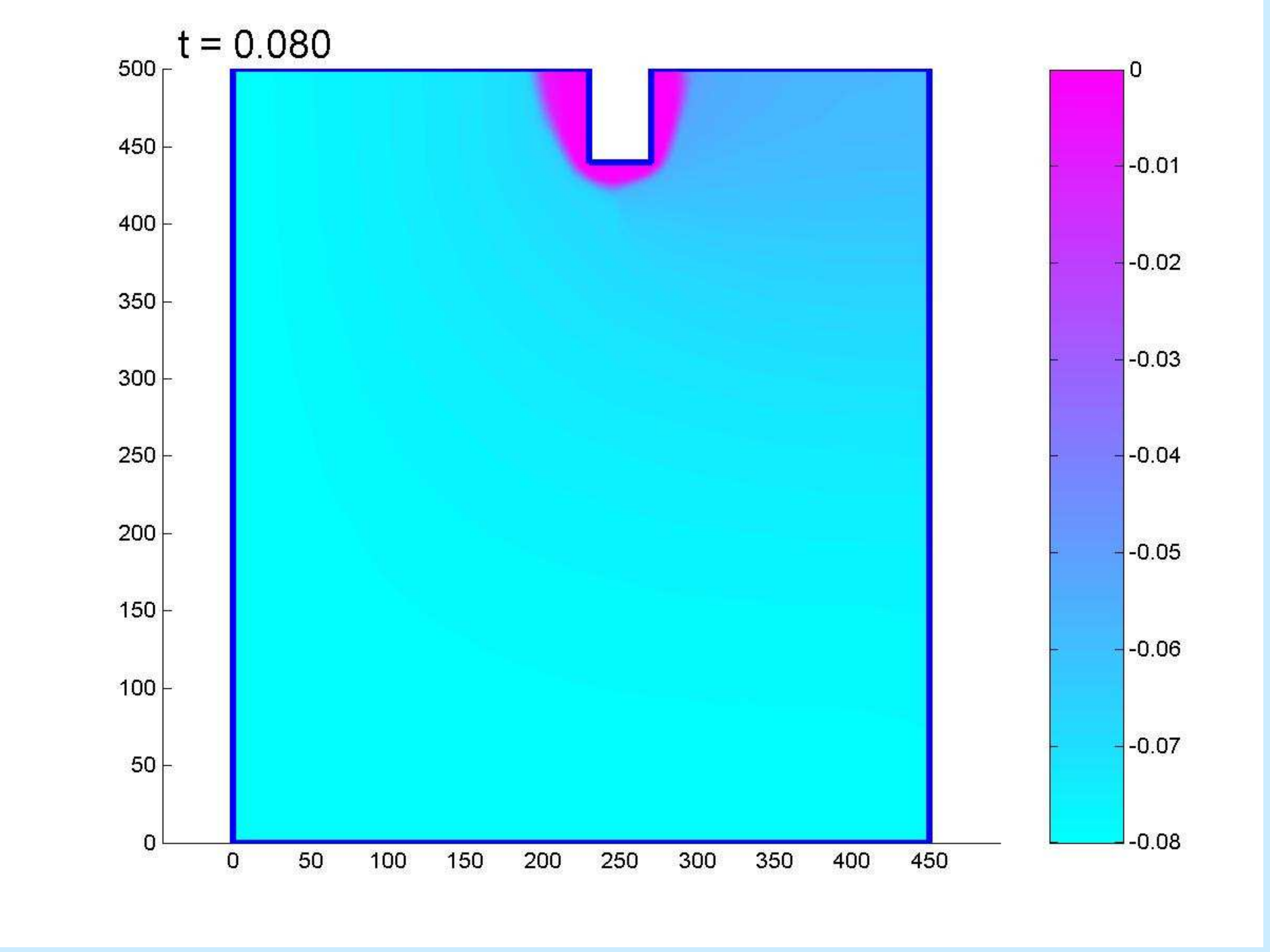
Cramp Pull – Deformed Mesh BFM

t = clamp pull (mm). Displacement amplification = 10^2

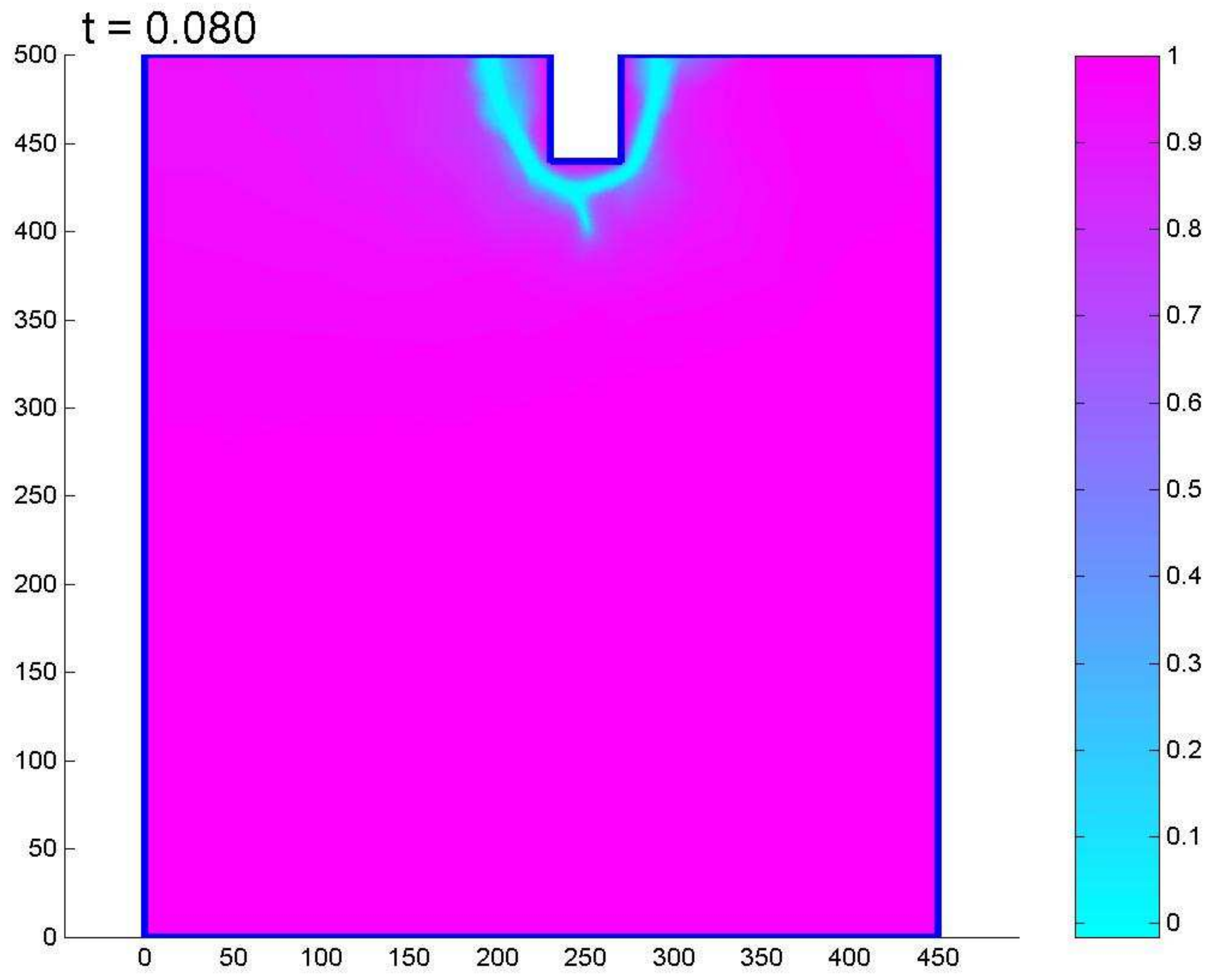


Cramp-pull - horizontal displacement BFM

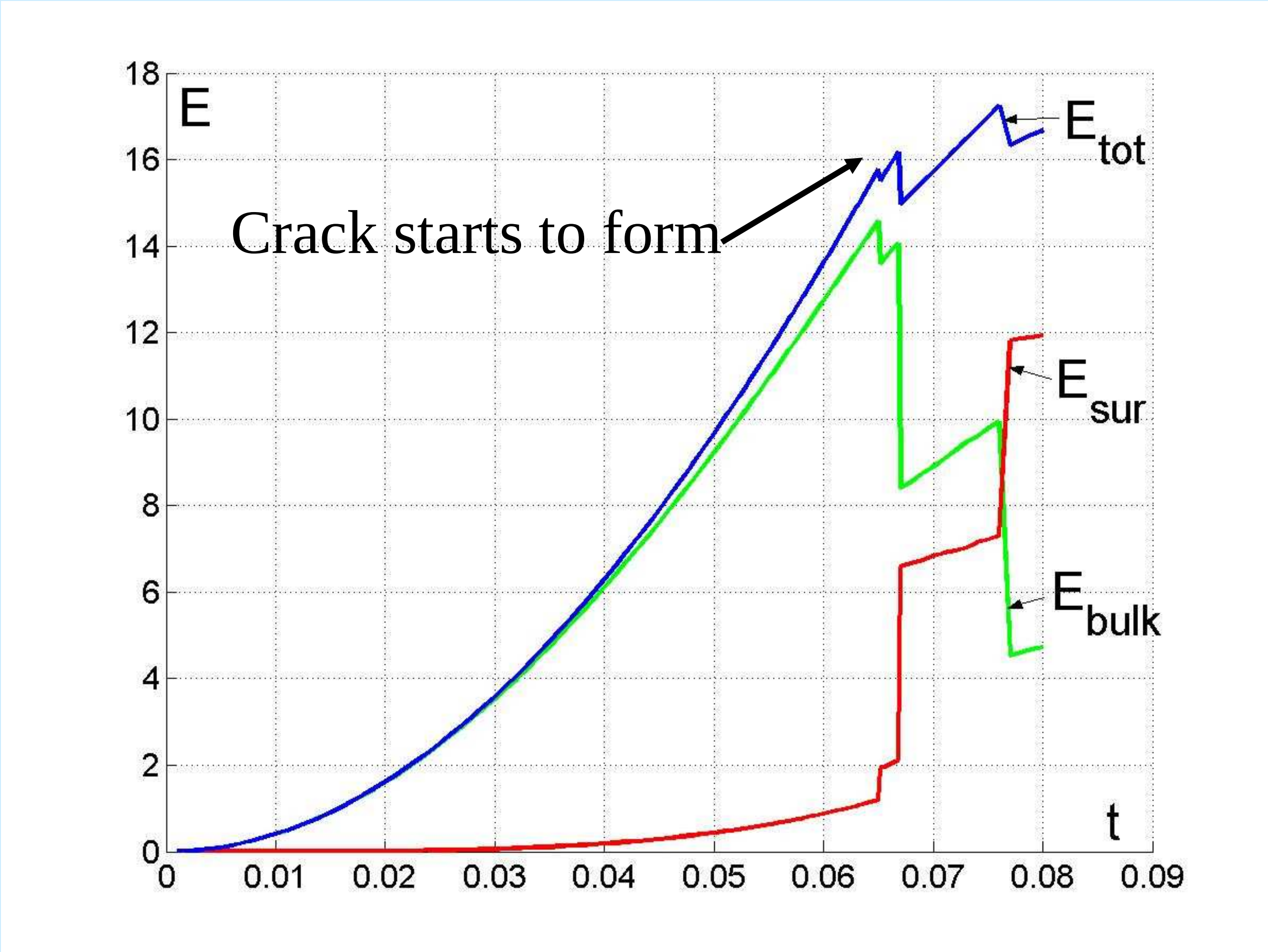
t = clamp pull (cm). Displacement (cm)



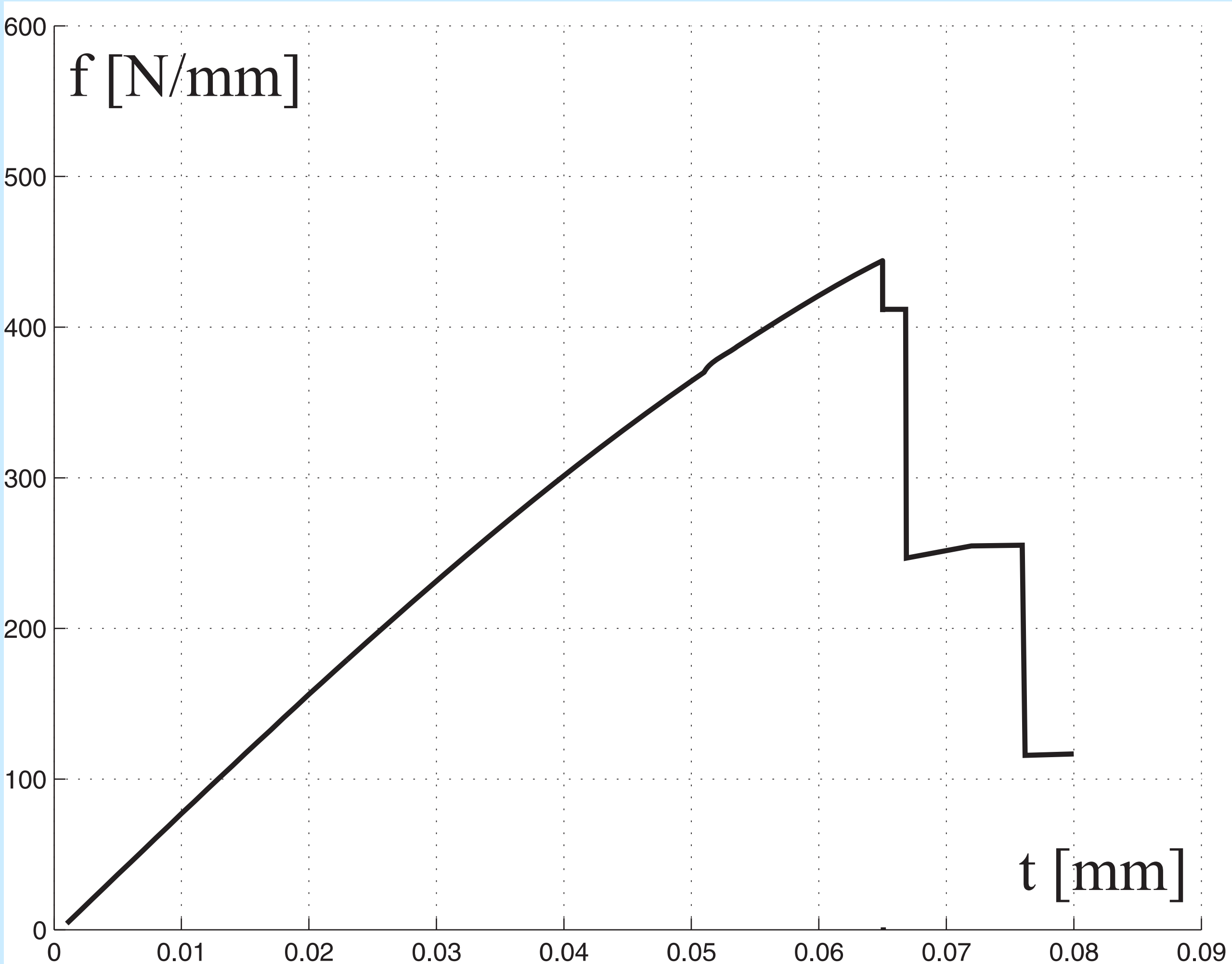
Cramp pull – damage field s BFM



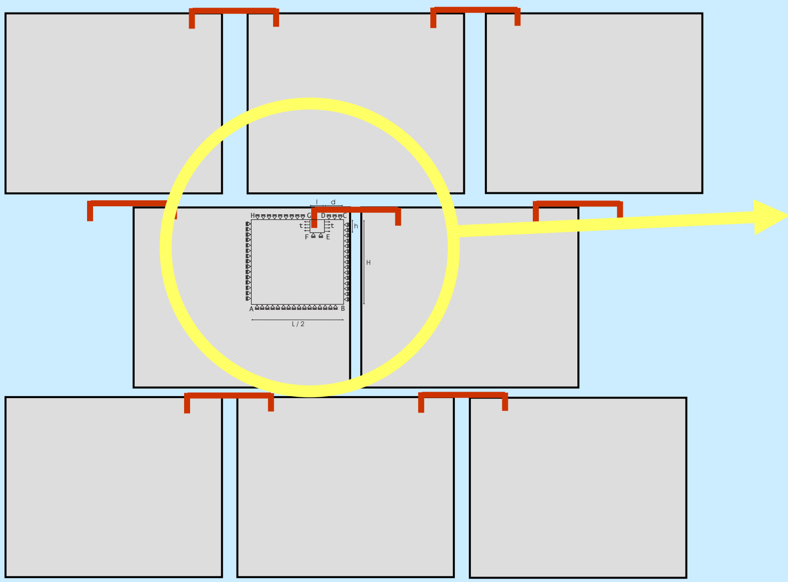
Cramp Pull – bulk and surface energy BFM

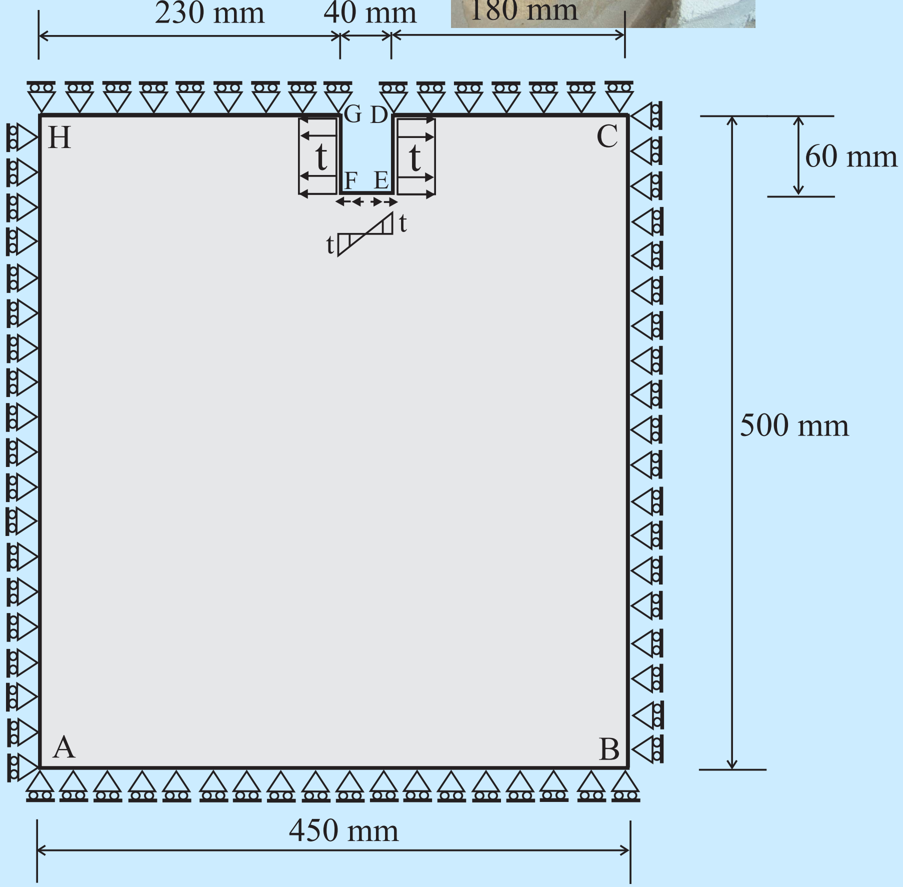


Cramp Pull – Force vs. displacement BFM



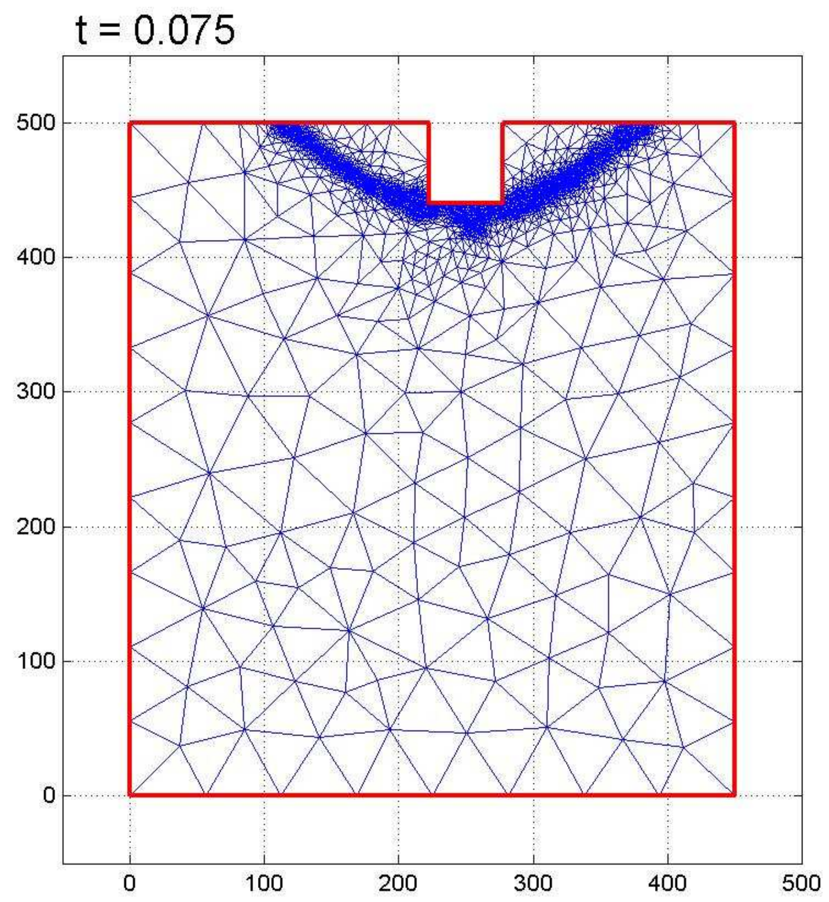
Numerical experiment. Cramp oxidation





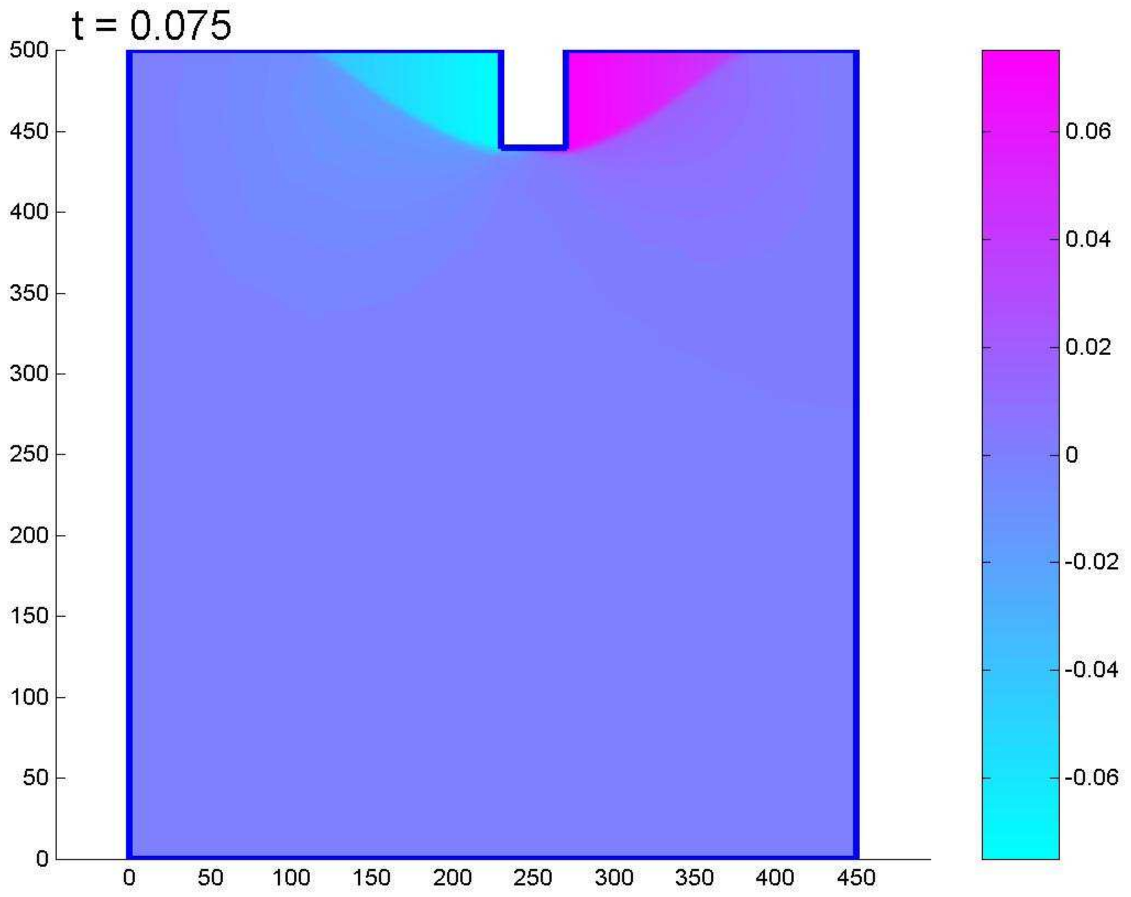
Cramp Oxidation– Deformed Mesh

t = clamp pull (mm). Displacement amplification = 10^2

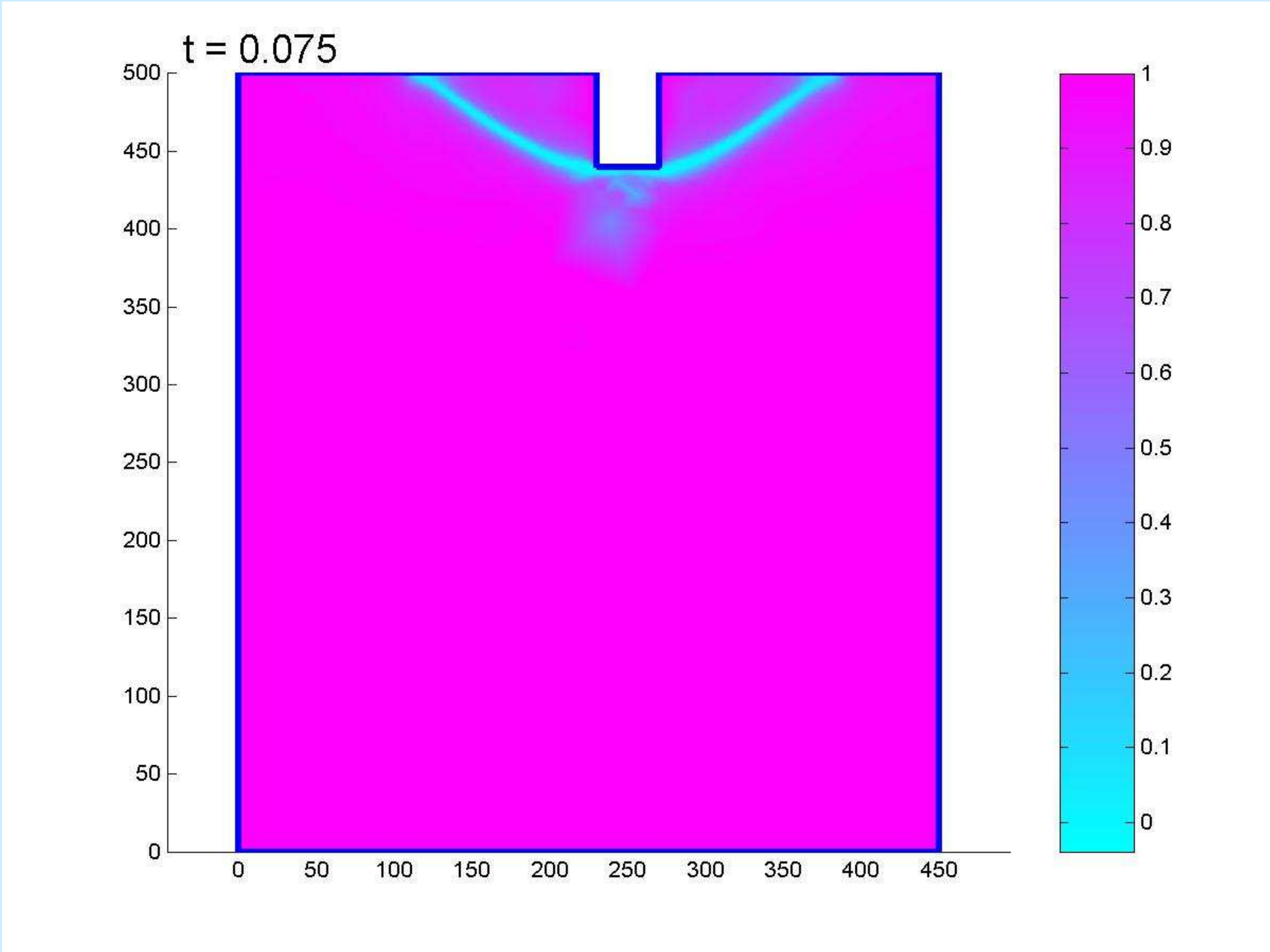


Cramp-oxidation - horizontal displacement

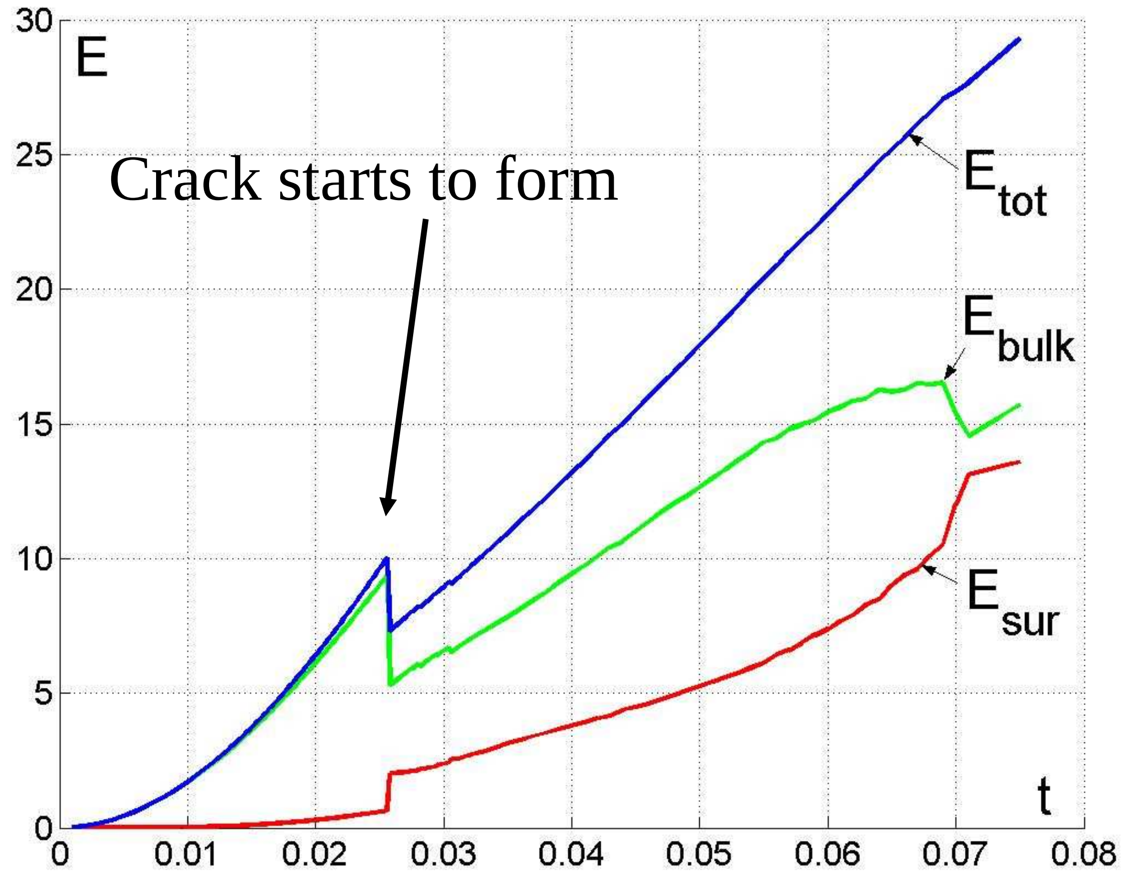
t = clamp pull (mm). Displacement (mm)



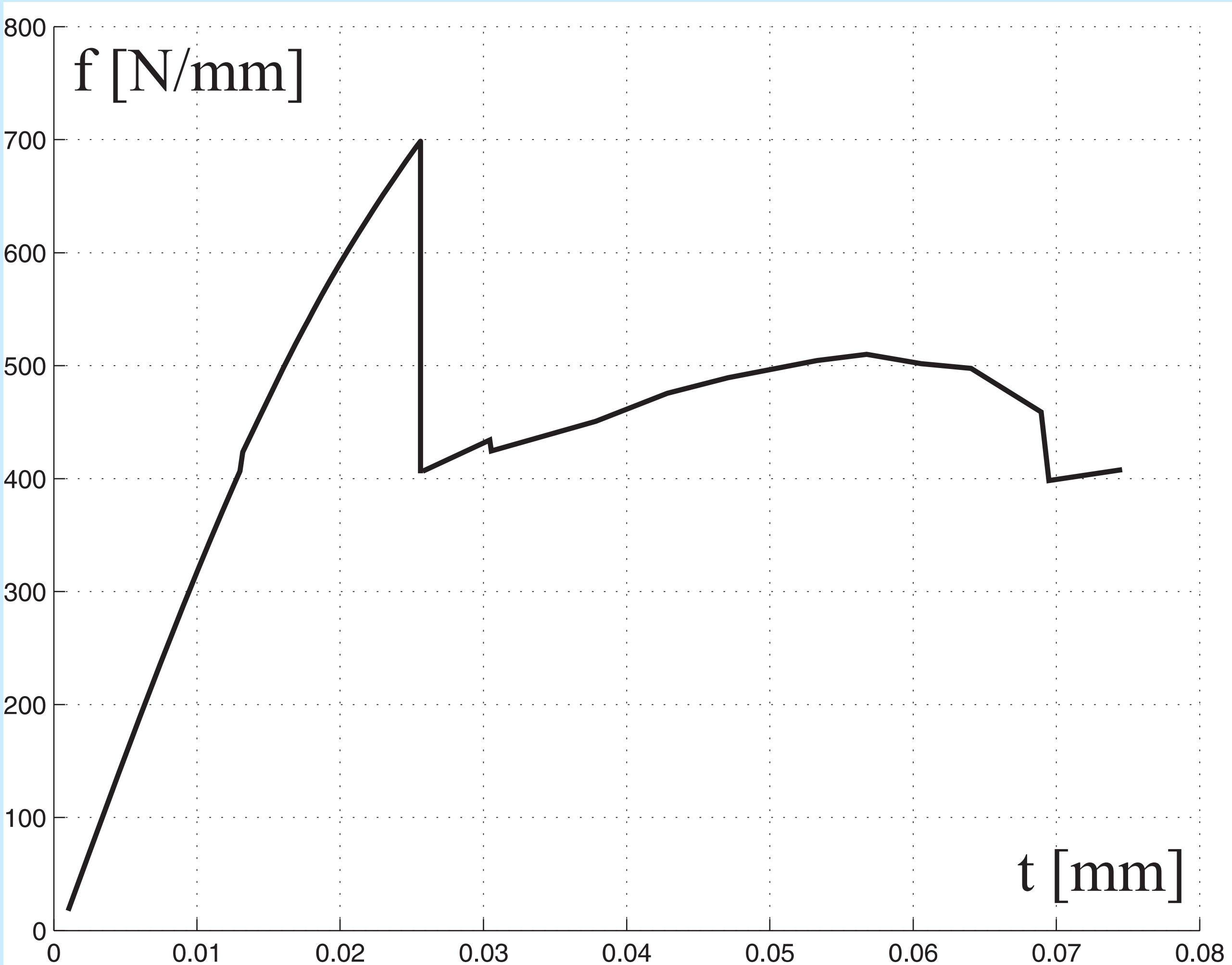
Cramp oxidation – damage field s



Cramp Oxidation – bulk and surface energy

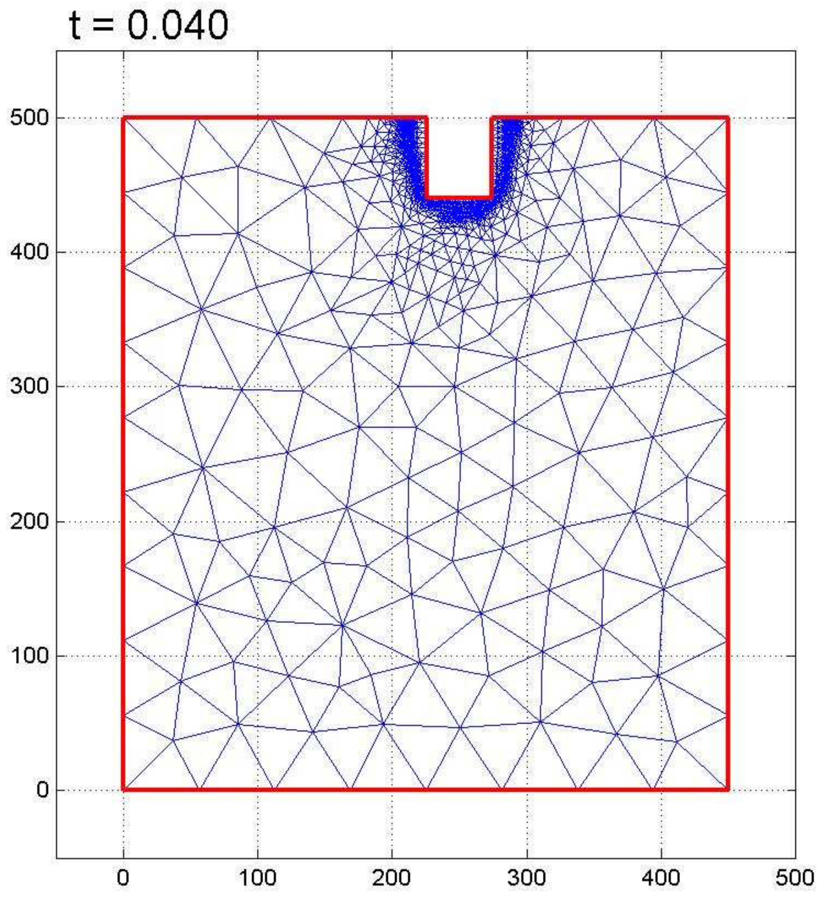


Cramp oxidation – Force vs. displacement



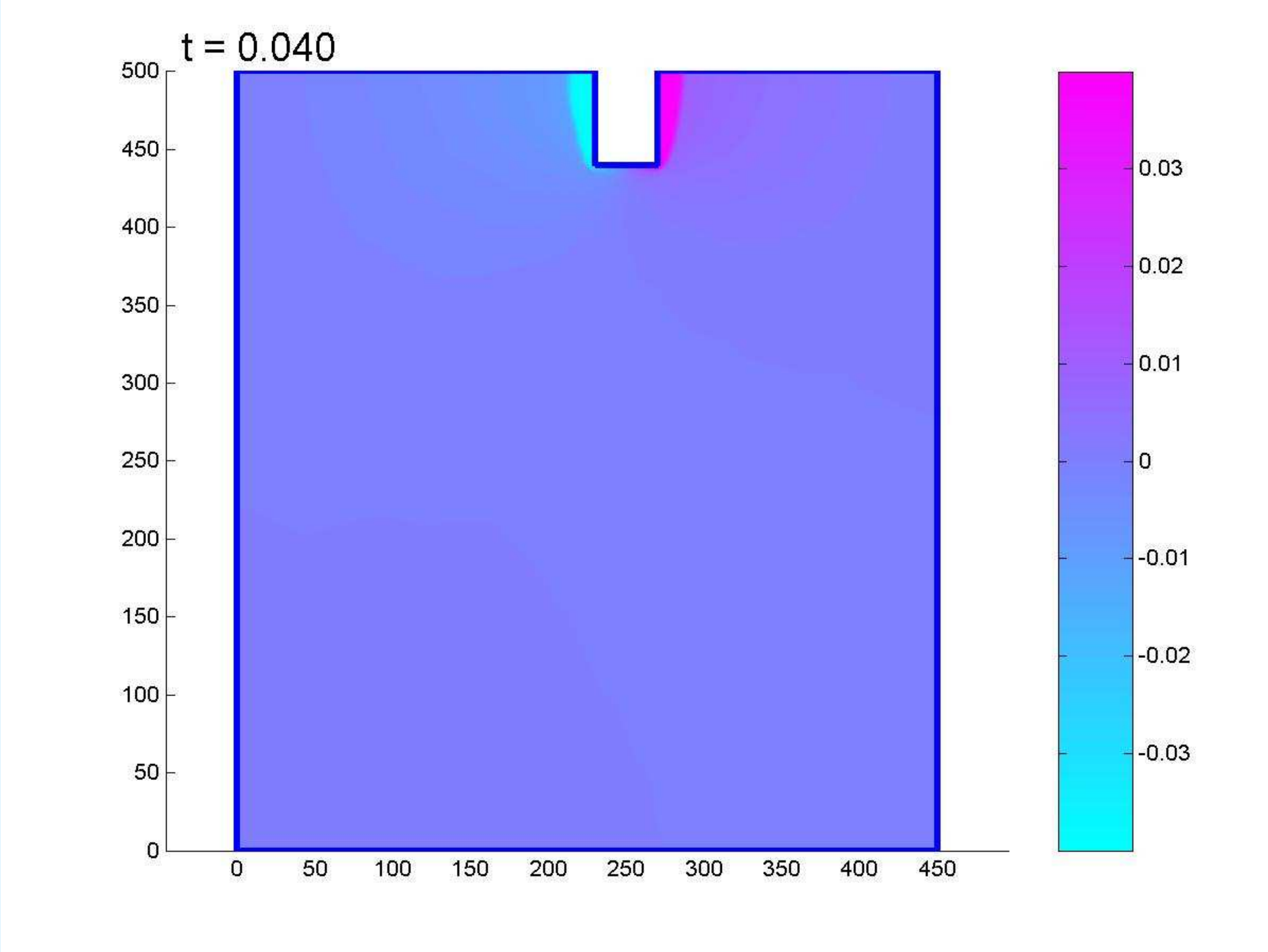
Cramp Oxidation– Deformed Mesh BFM

t = clamp pull (cm). Displacement amplification = 10^2

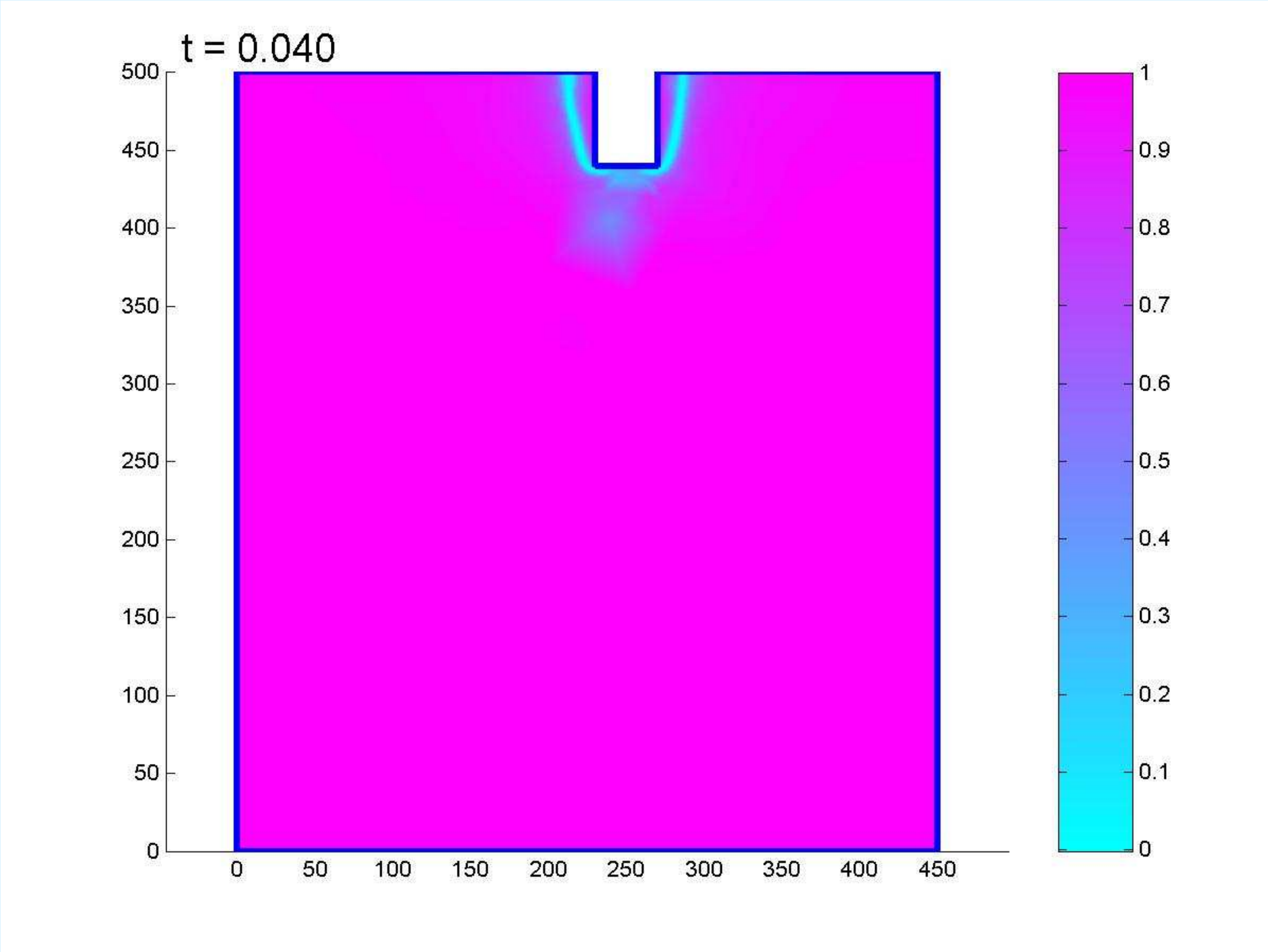


Cramp-oxidation - horizontal displacement BFM

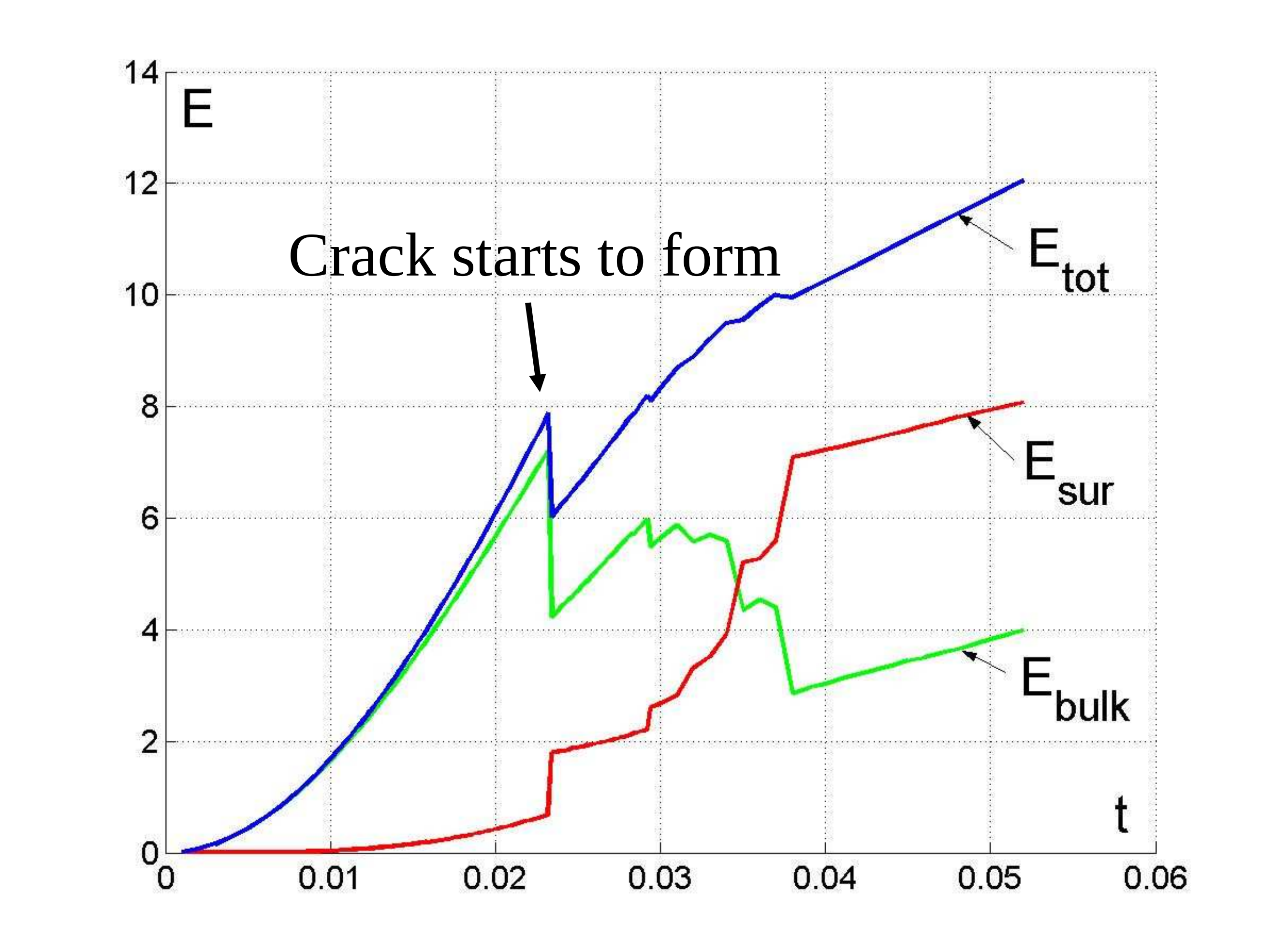
t = clamp pull (cm). Displacement (cm)



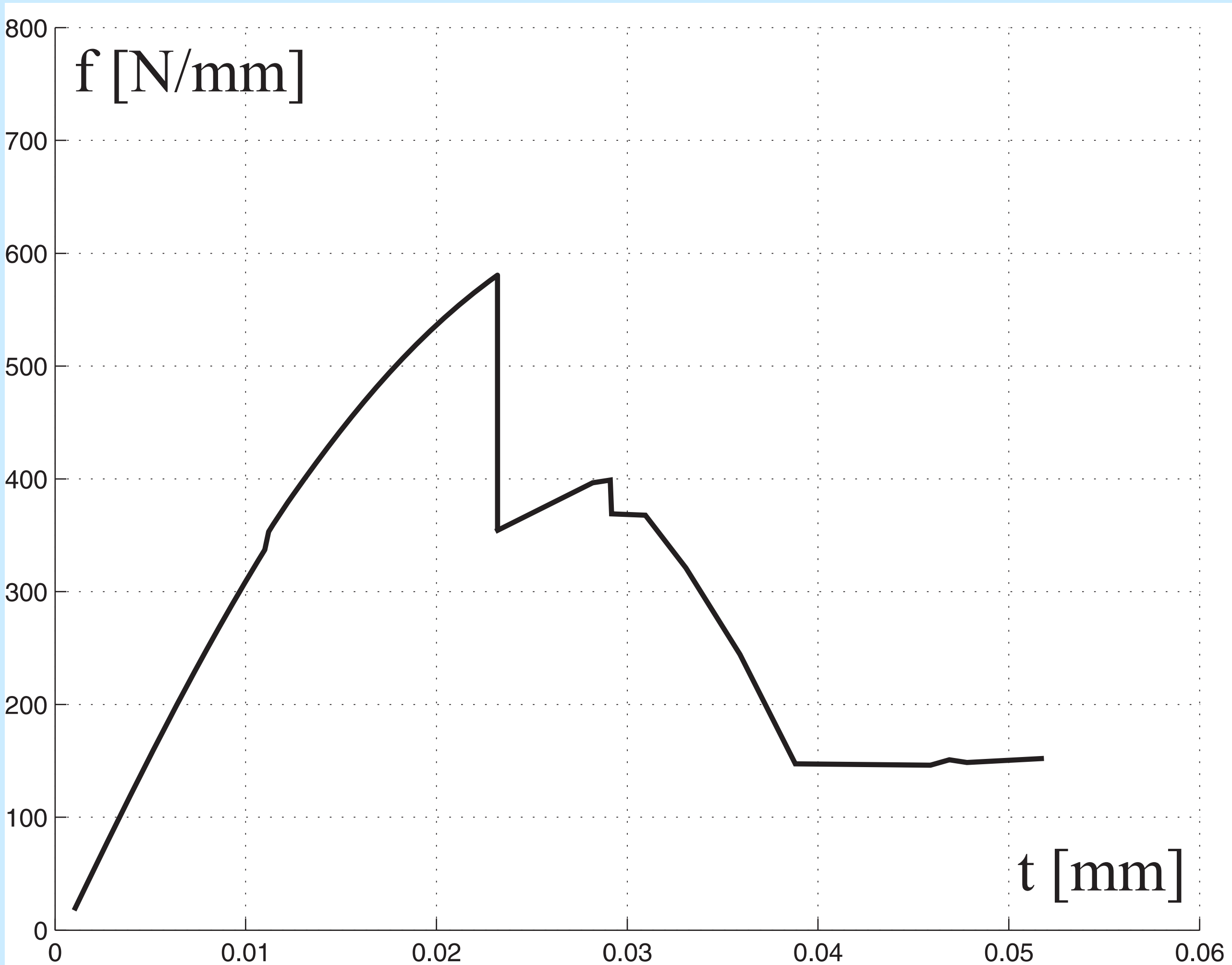
Cramp oxidation – damage field s BFM



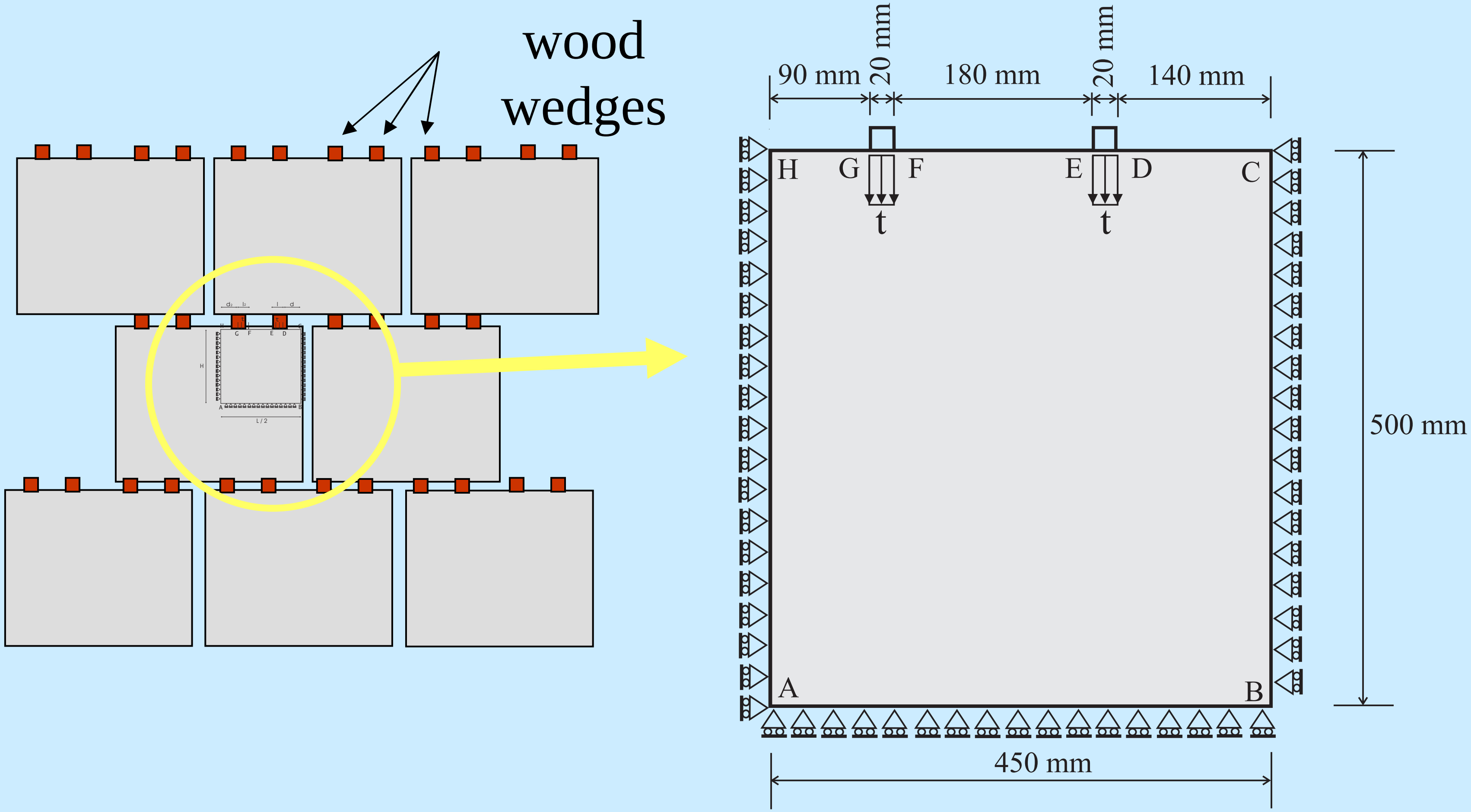
Cramp Oxidation – bulk and surface energy BFM



Cramp oxidation – Force vs. displacement BFM

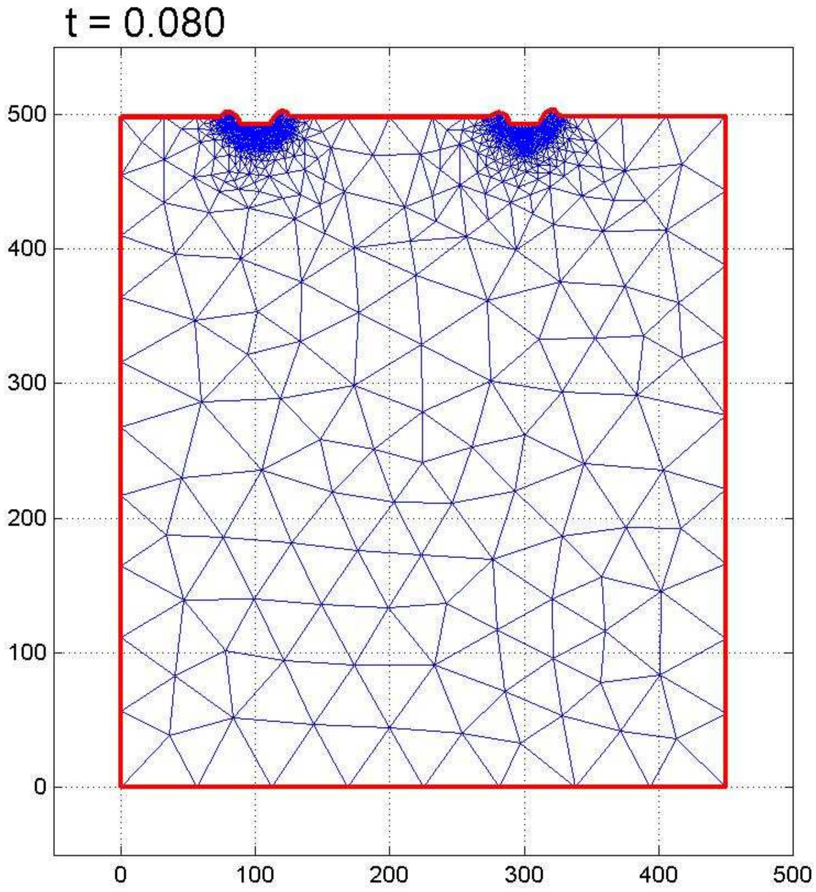


Numerical experiment.
Oak wood wedge contact



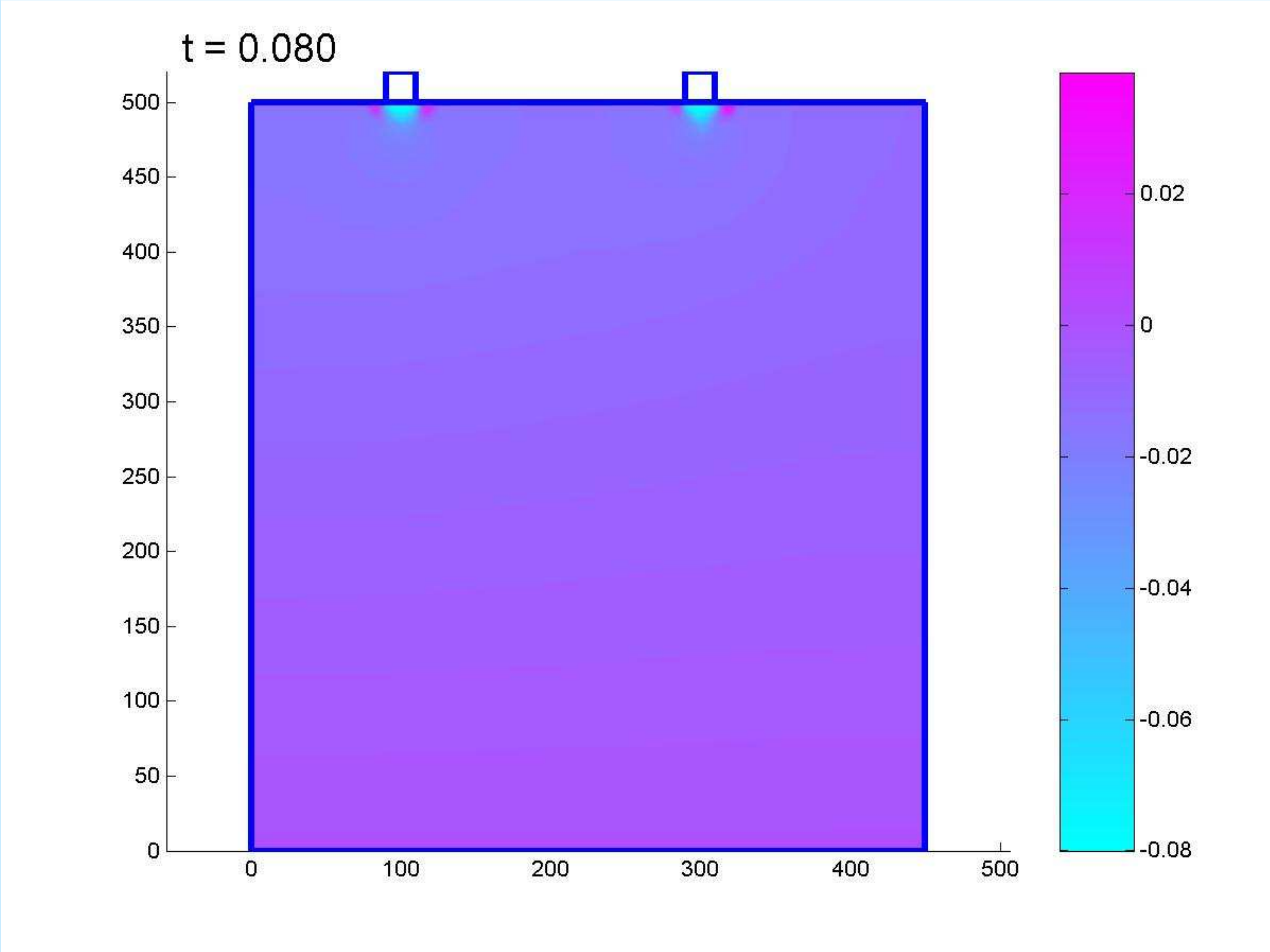
Contact – Deformed Mesh

t = contact dispacement (mm). Displacement amplification = 10^2

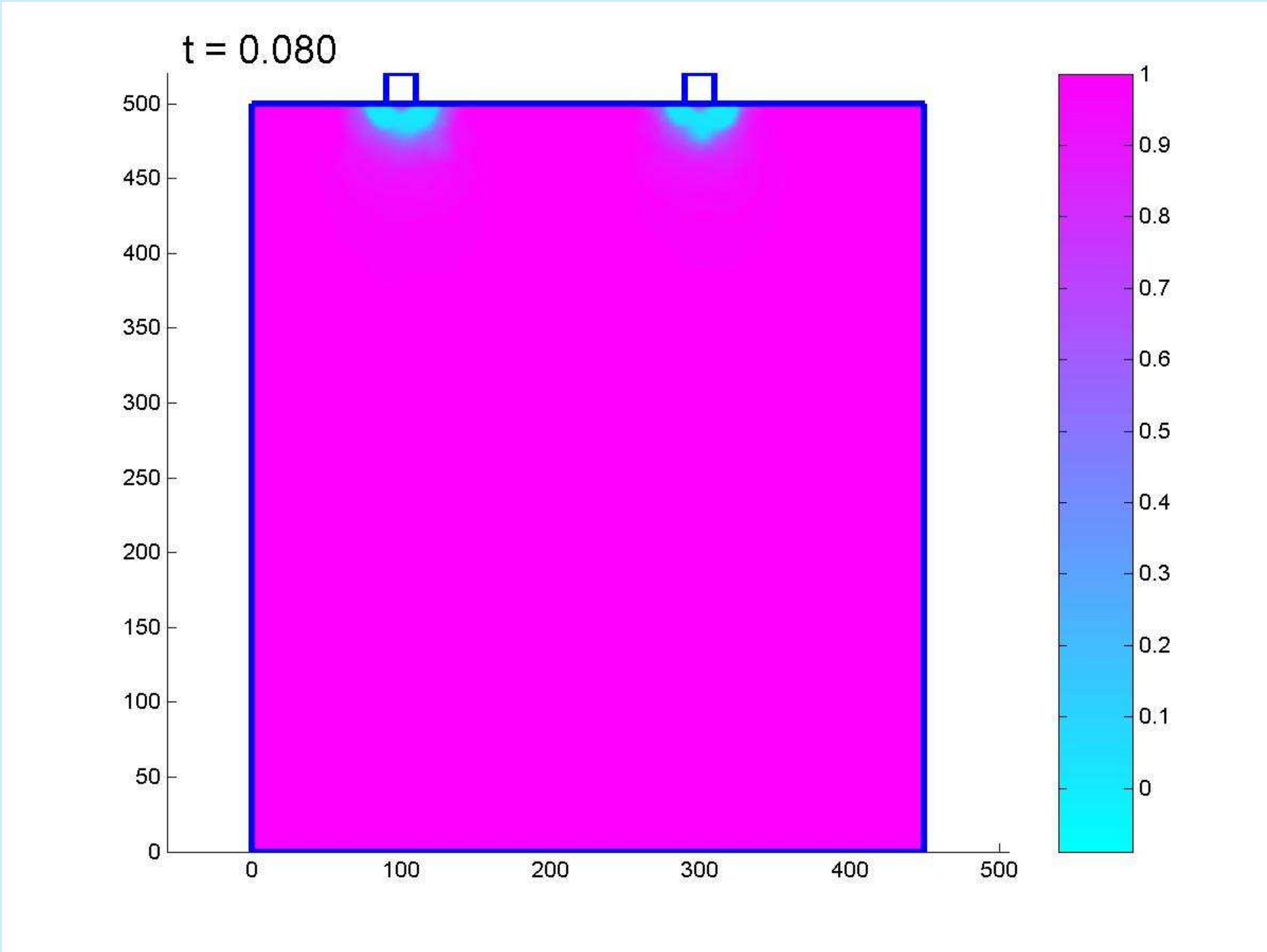


Contact - vertical displacement

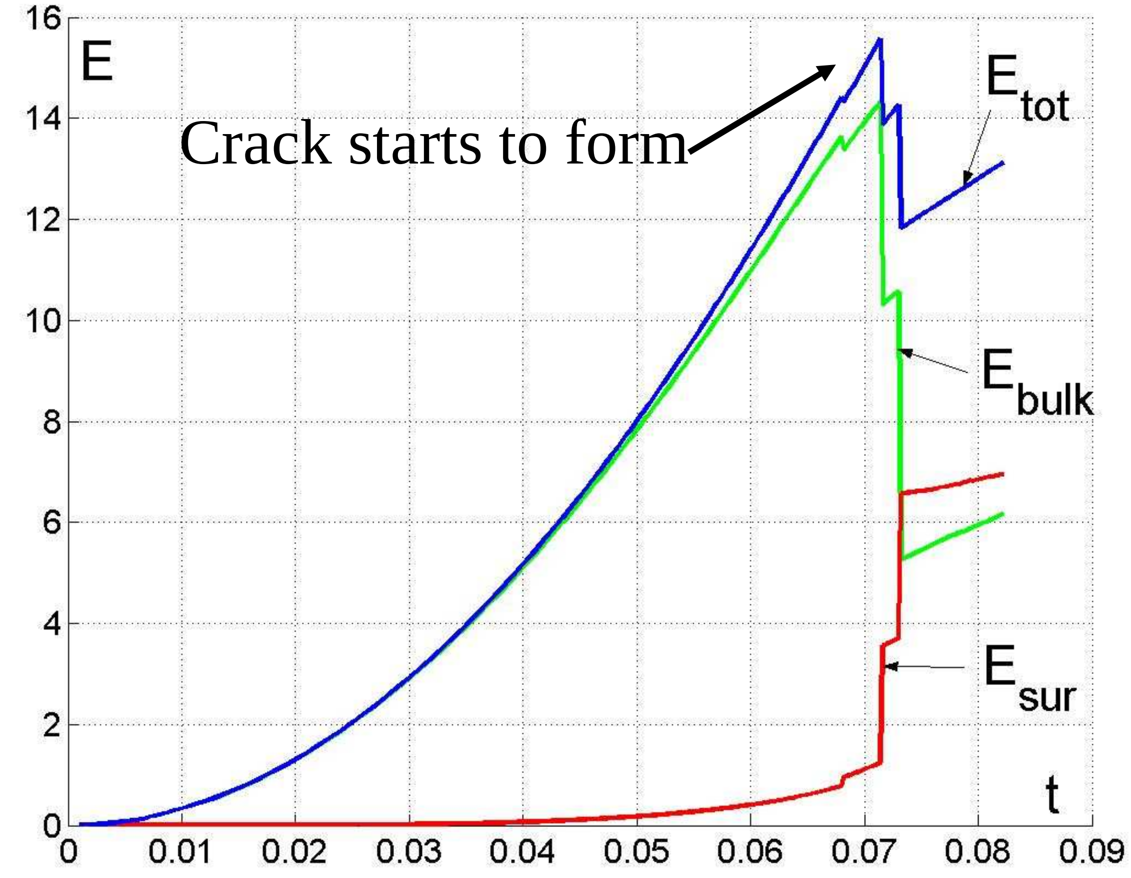
t = contact displacement (mm). Displacement (mm)



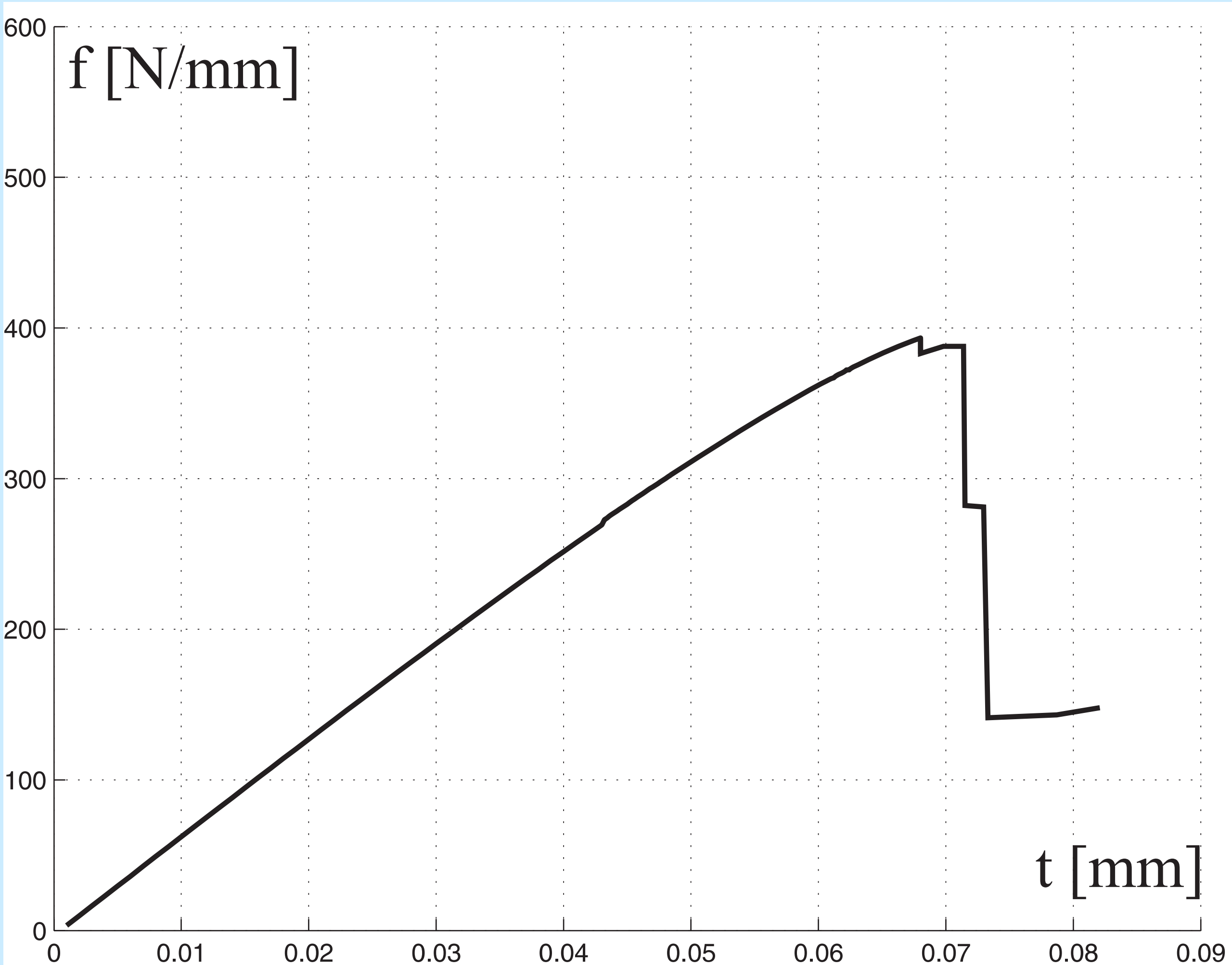
Contact – damage field s



Contact – bulk and surface energy

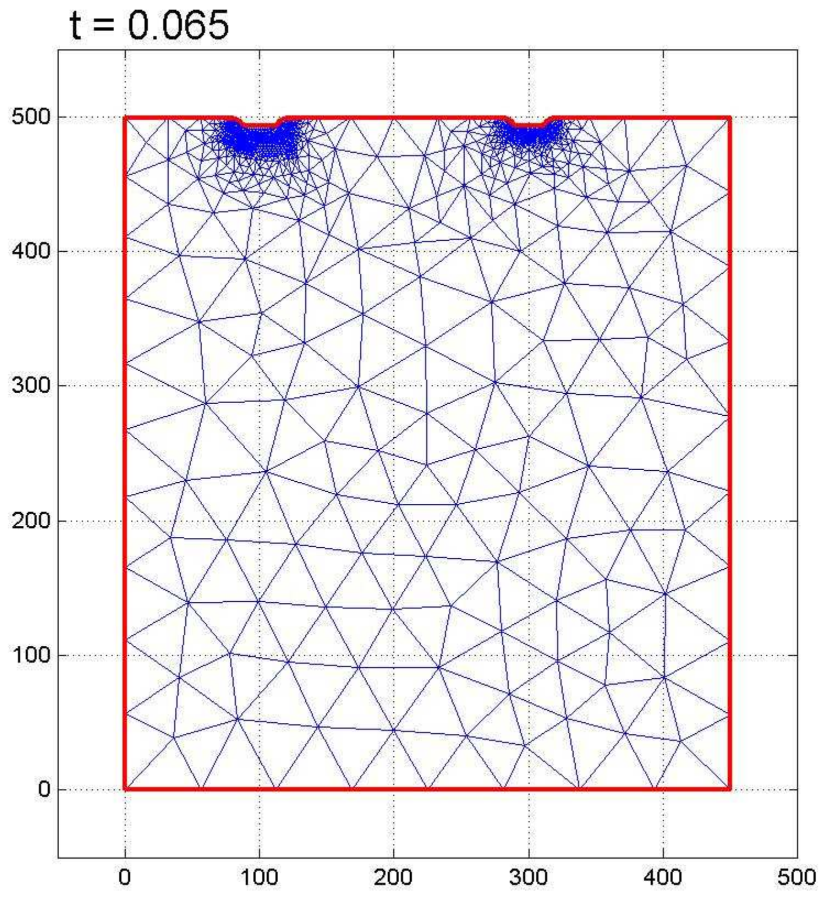


Contact – Force vs. displacement



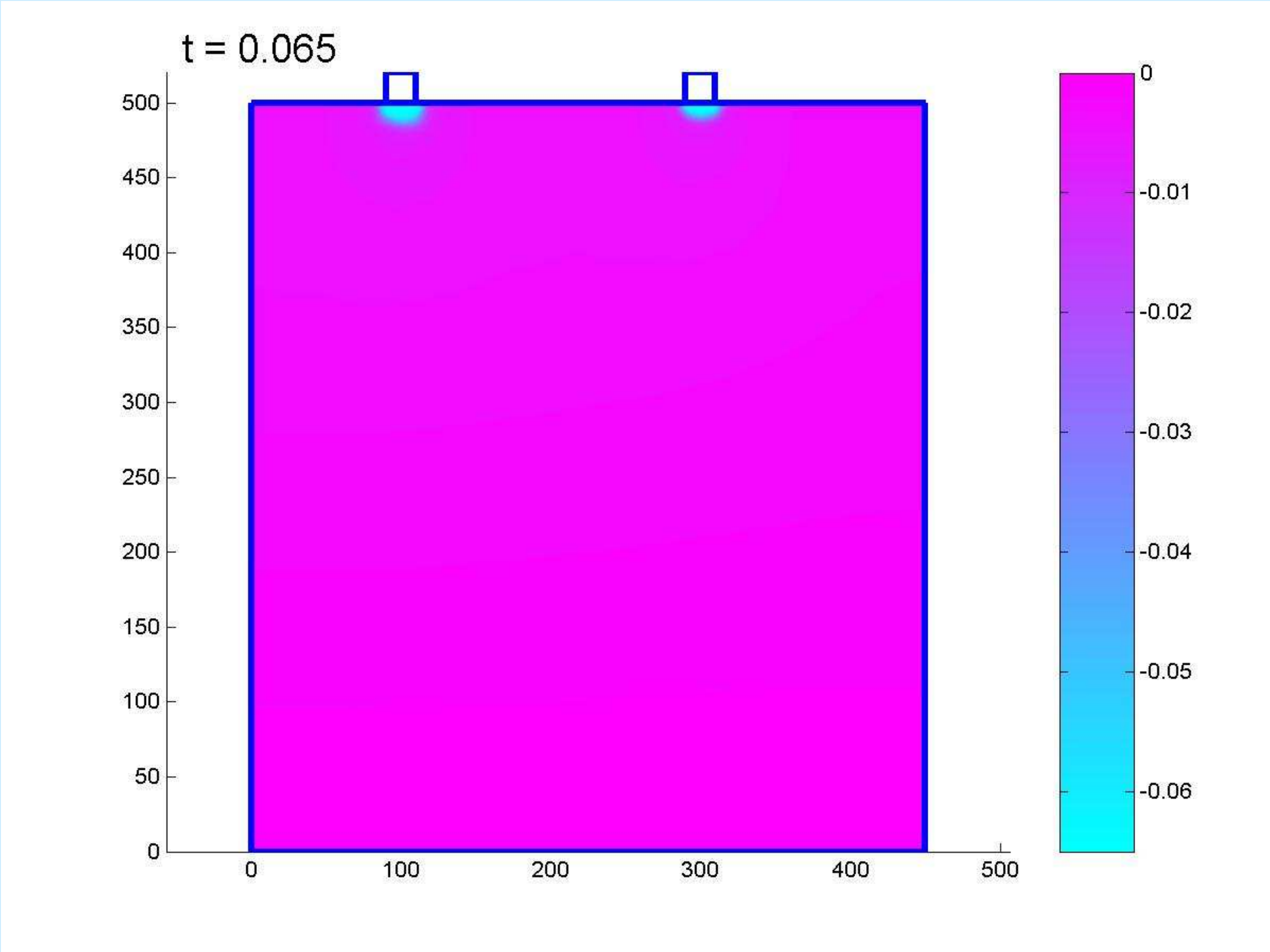
Contact – Deformed Mesh BFM

t = contact dispacement (mm). Displacement amplification = 10^2

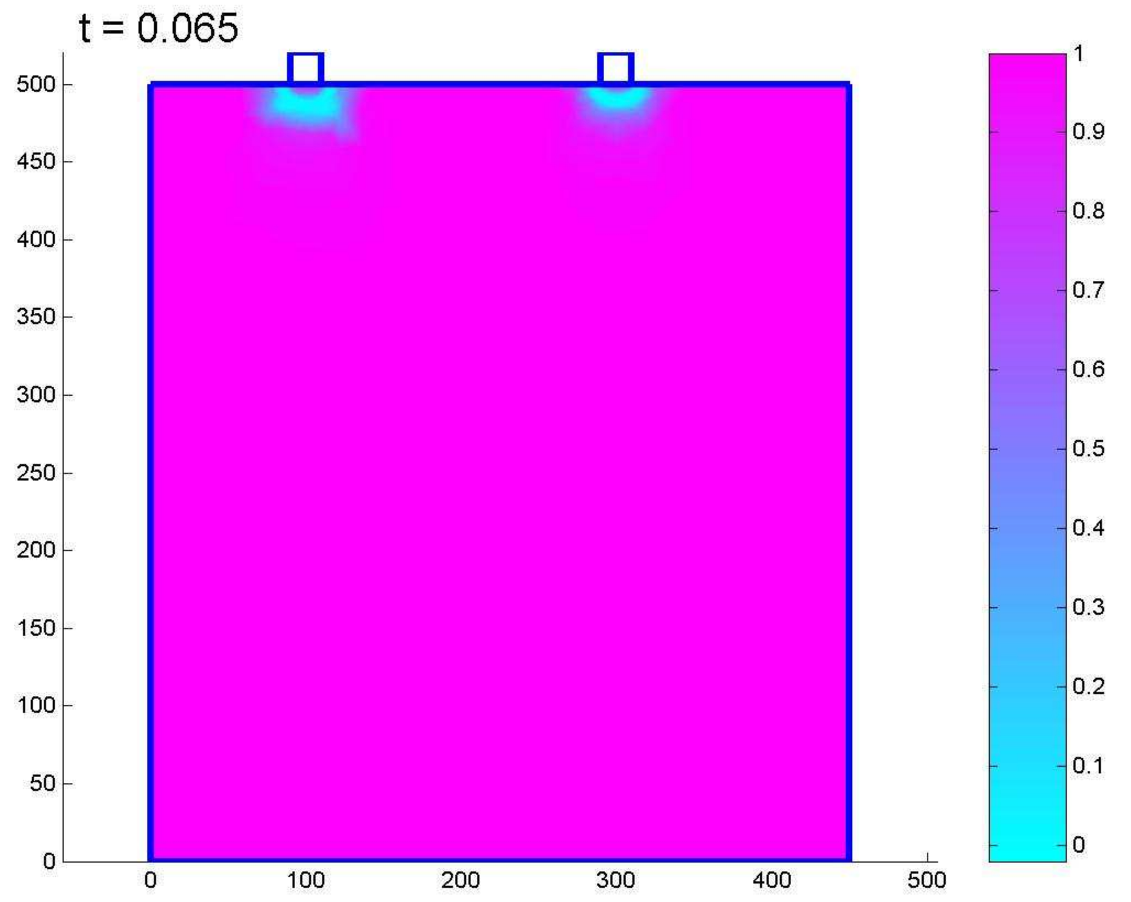


Contact - vertical displacement BFM

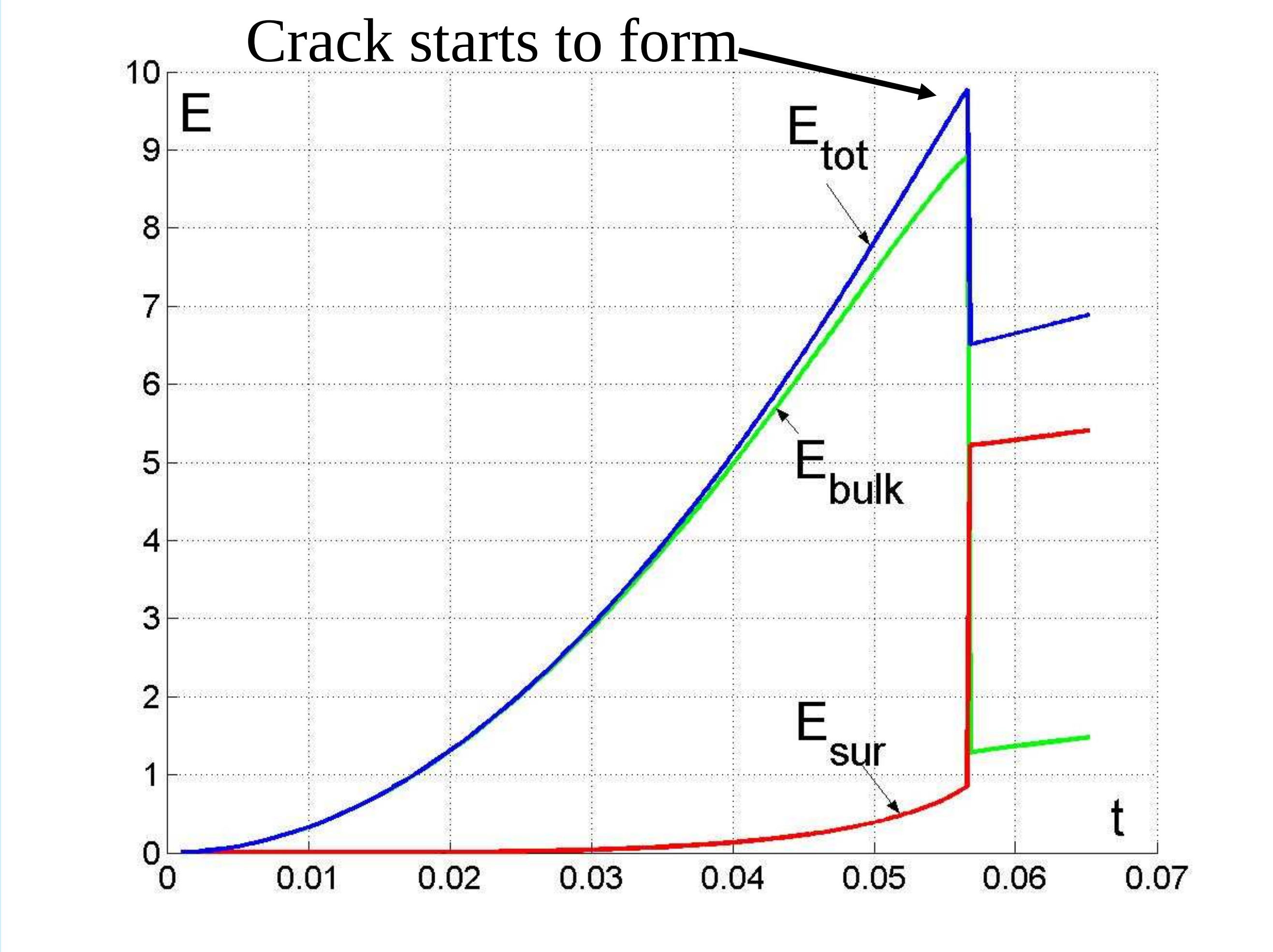
t = contact displacement (mm). Displacement (mm)



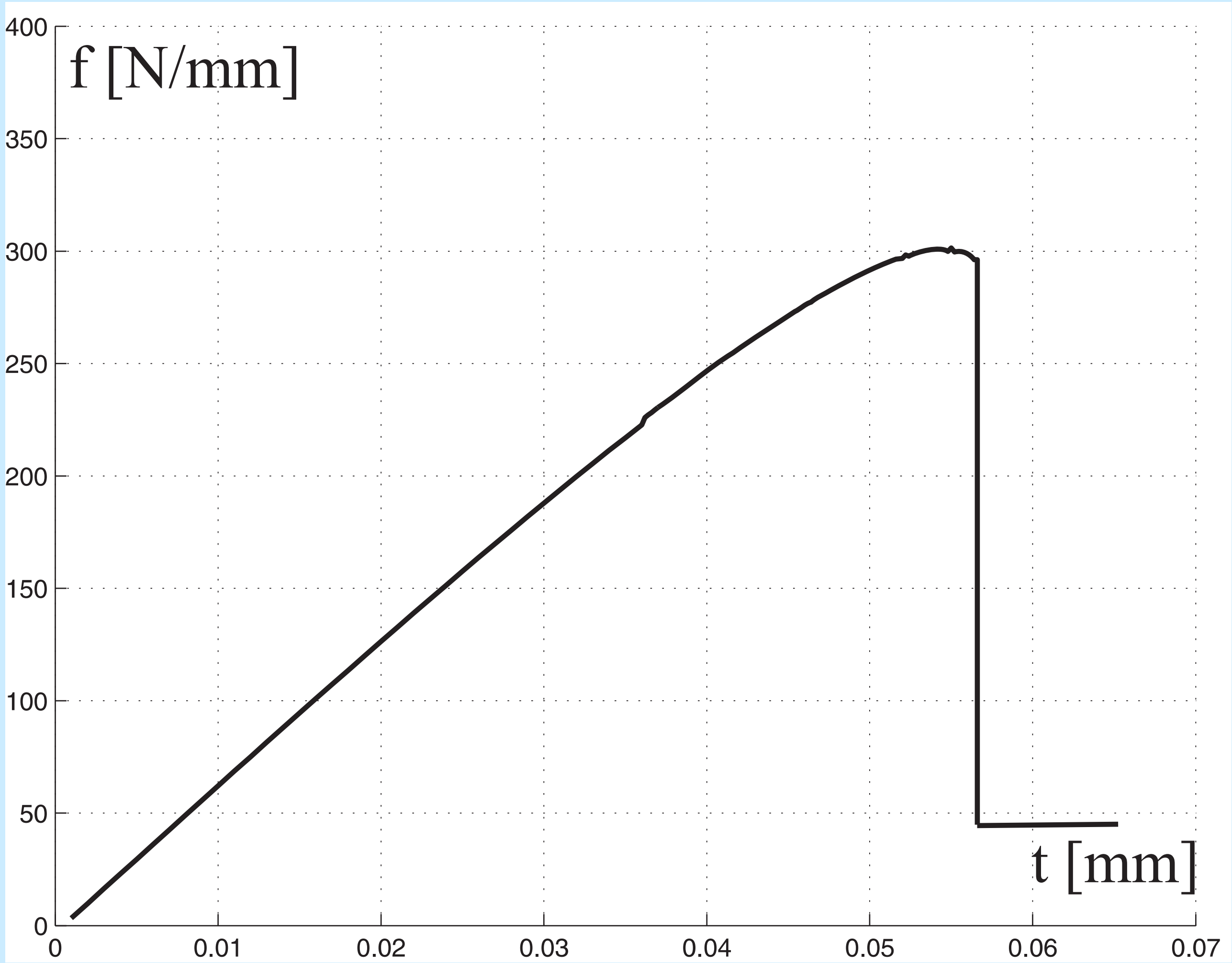
Contact – damage field s BFM



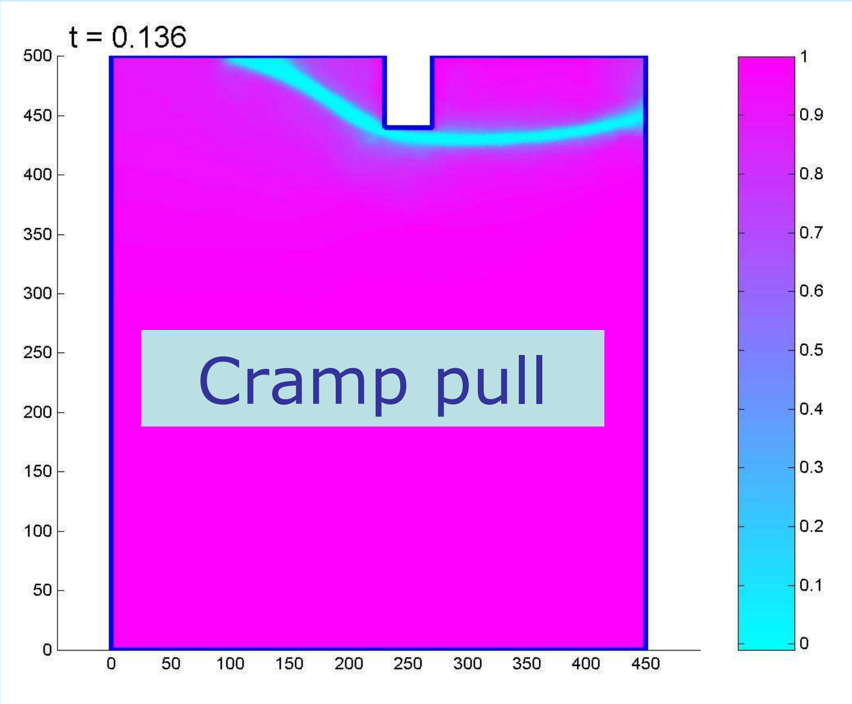
Contact – bulk and surface energy BFM

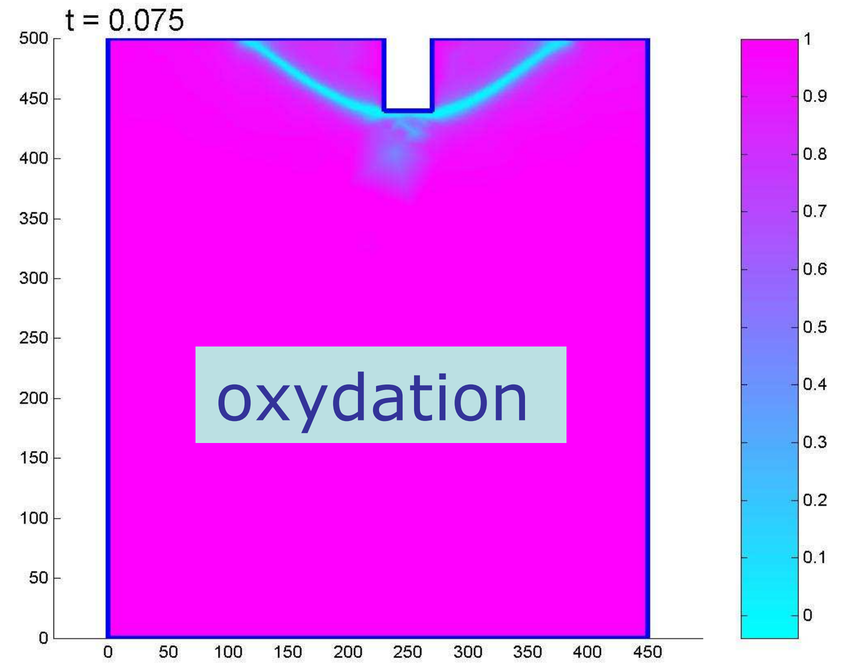


Contact – Force vs. displacement BFM

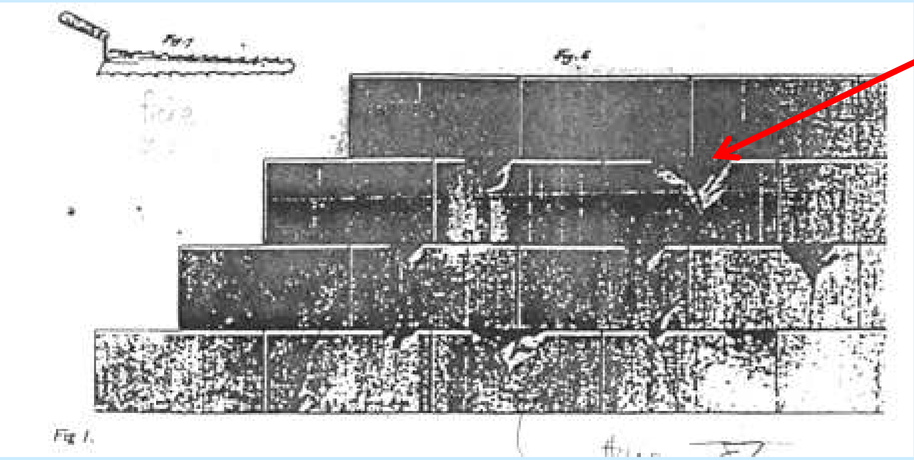


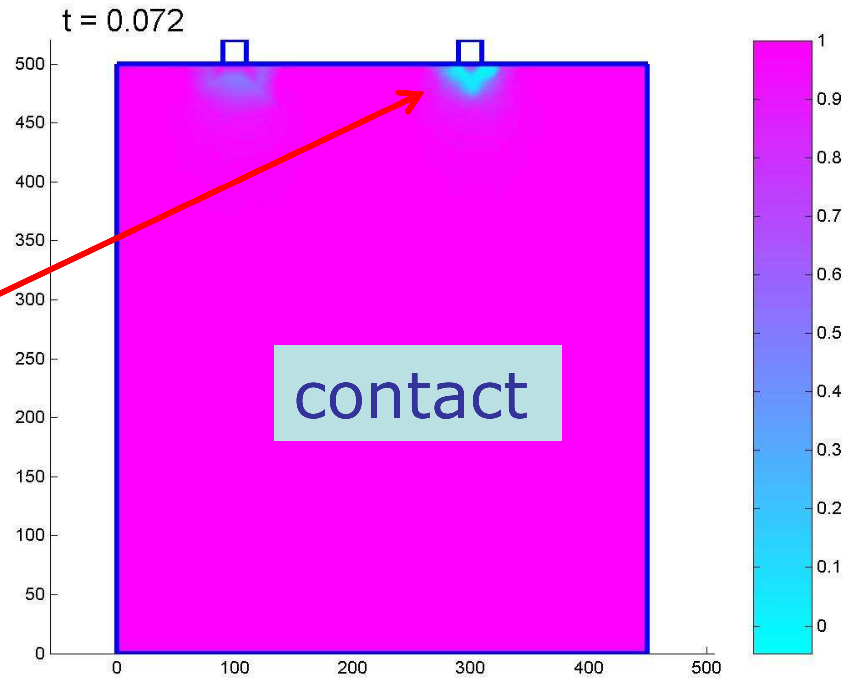
Quantitative analysis. The pylons must have failed during construction (as documented)



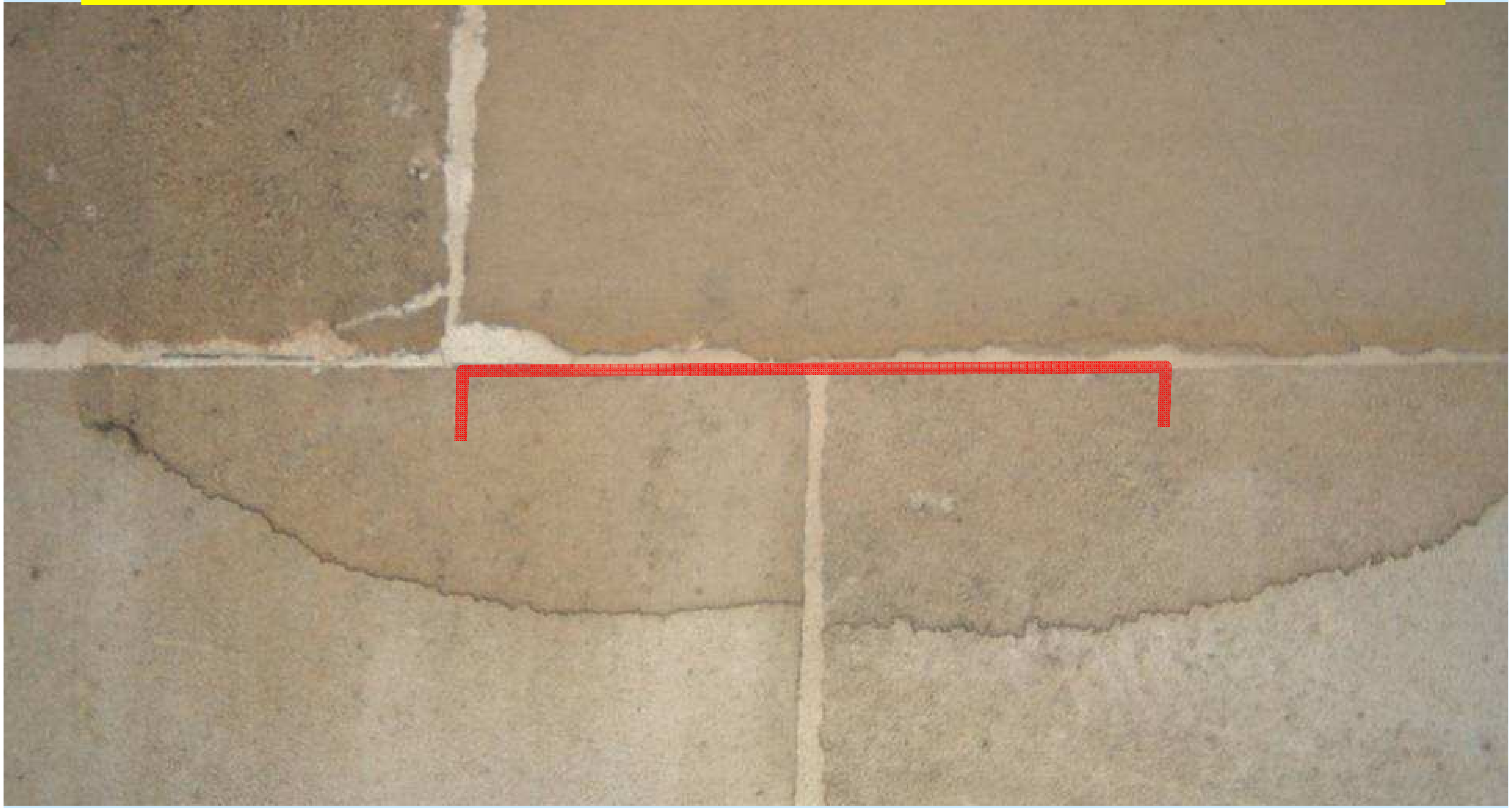


Crack paths produced by different causes

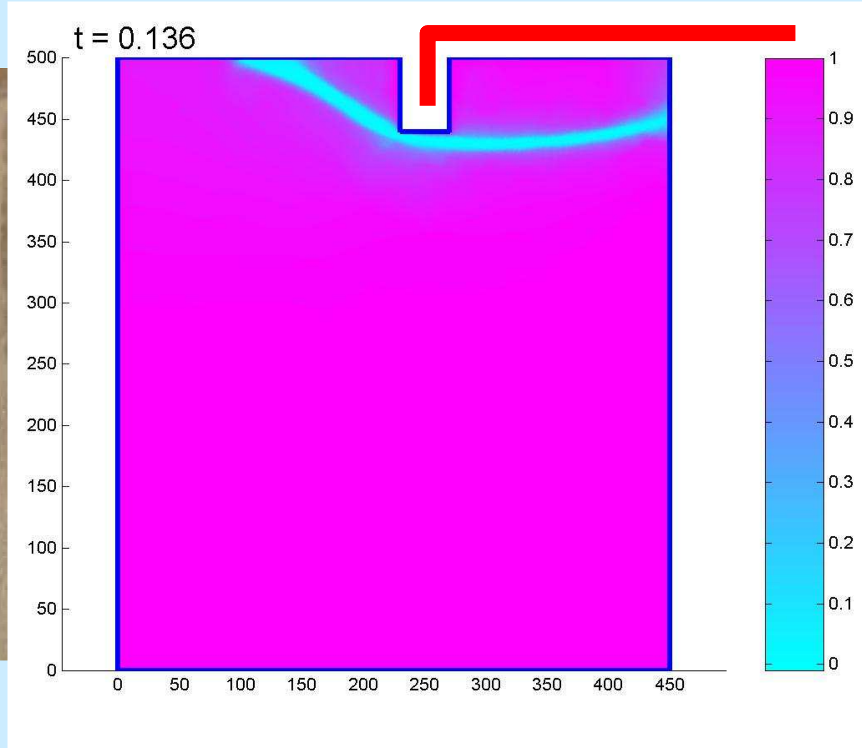




FRACTURE SHAPE – CRAMP PULL

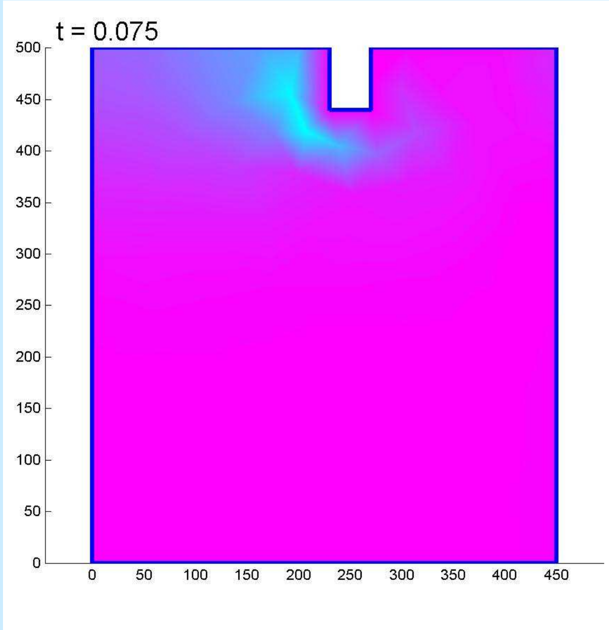


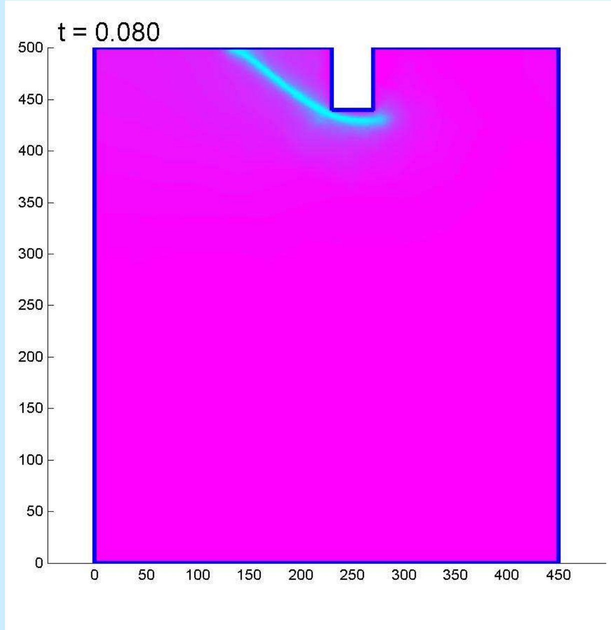
Fracture shape reproduced by simulation

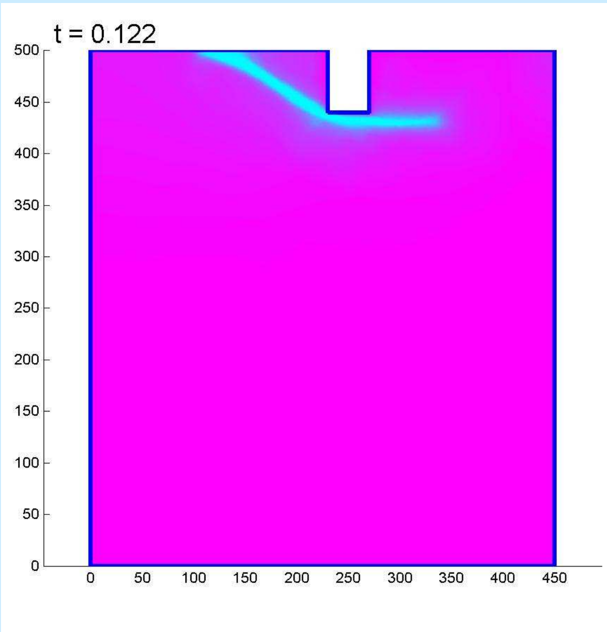


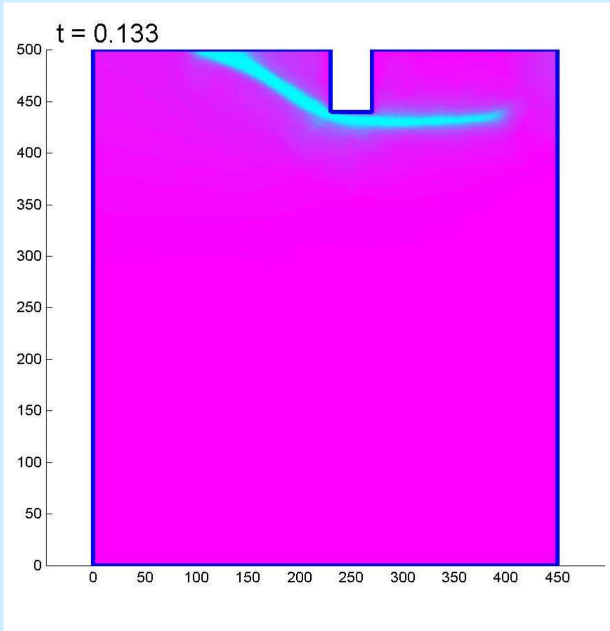
Crack evolution under

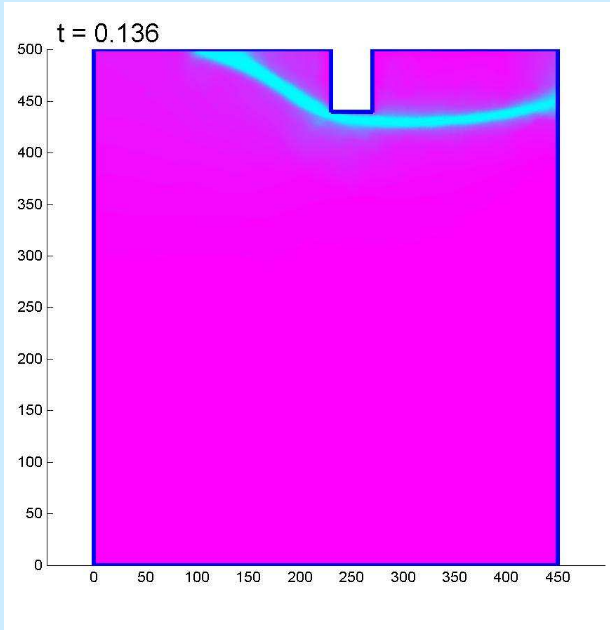
cramp pull



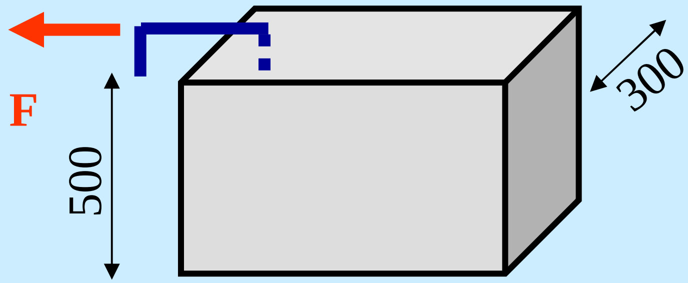








Quantitative results



Fracture energy $\gamma = 25 \cdot 10^{-3}$ N/mm

PEAK LOAD (per unit panel thickness): 500 N/mm

For a panel 300mm-thick

F = 500 x 300 = 150000 N

If the panel is 500 mm high, hoop stress producing rupture is

σ = 150000/(300 x 500) = 1N/mm^2

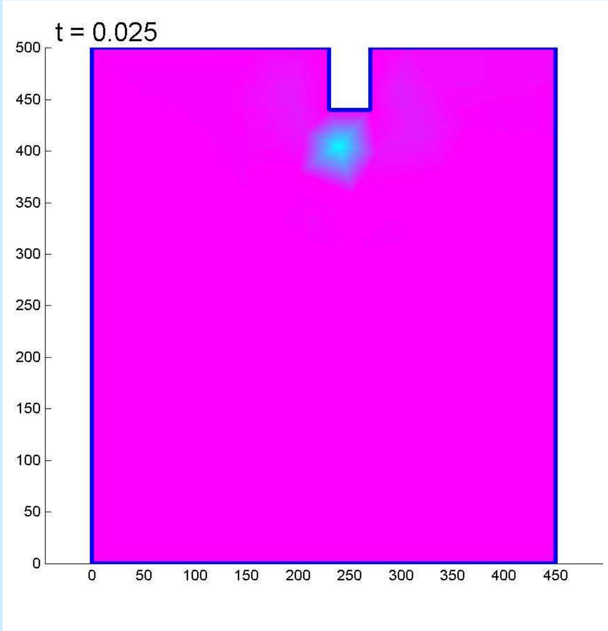
Such a value is compatible with the structural analysis of the vaults

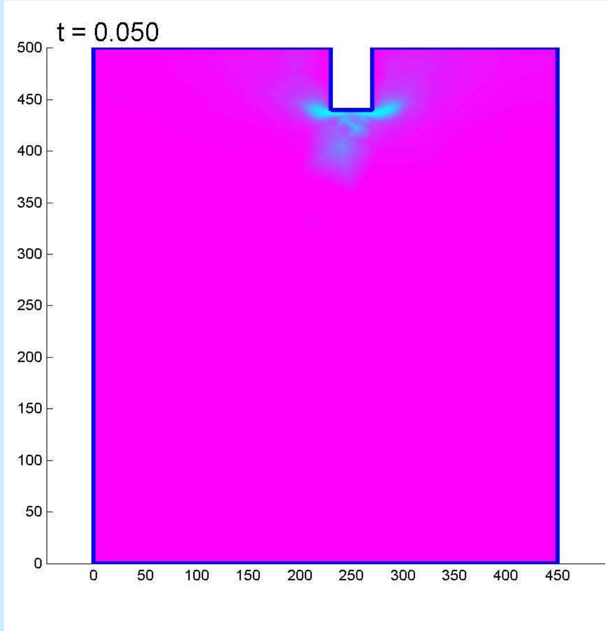
Maximum displacement under peak load

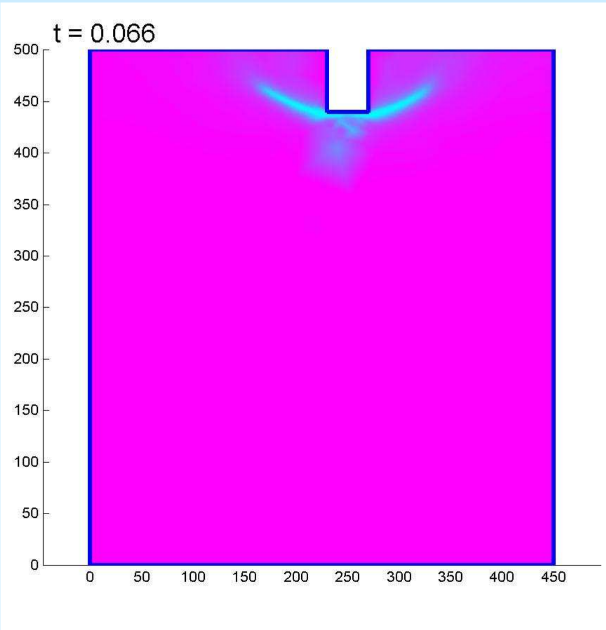
δ = 0.07 mm **BRITTLE RUPTURE**

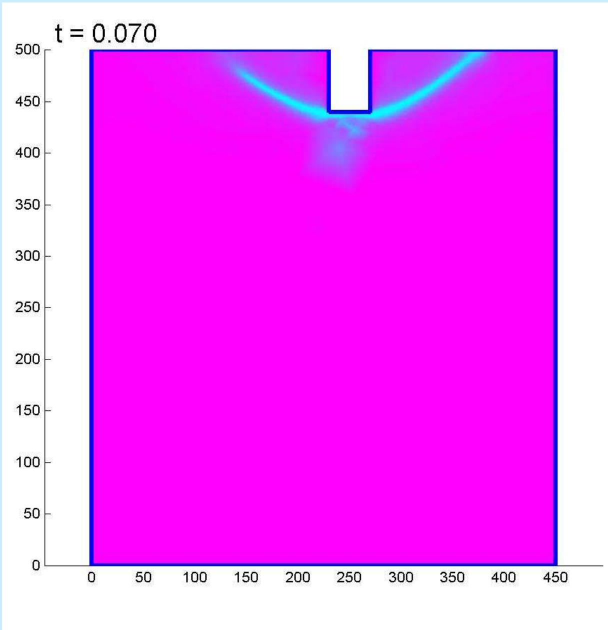
Crack evolution under

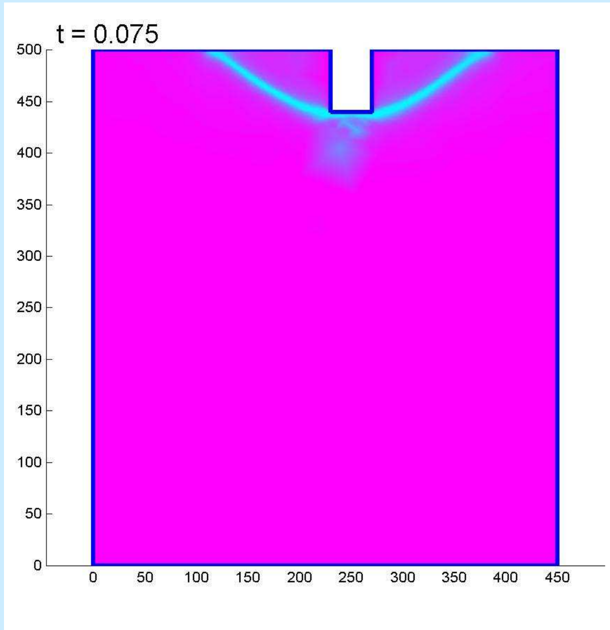
cramp oxidation









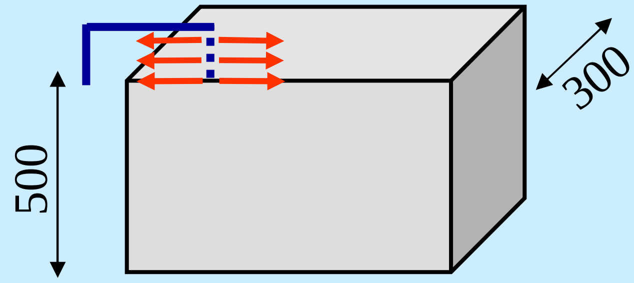


Quantitative results

PEAK LOAD (per unit panel thickness): 250 N/mm

Corresponding displacement

t = 0.02 mm

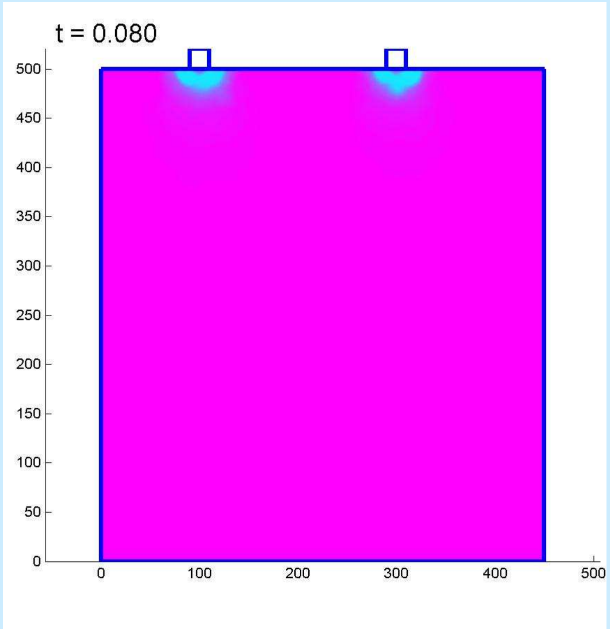


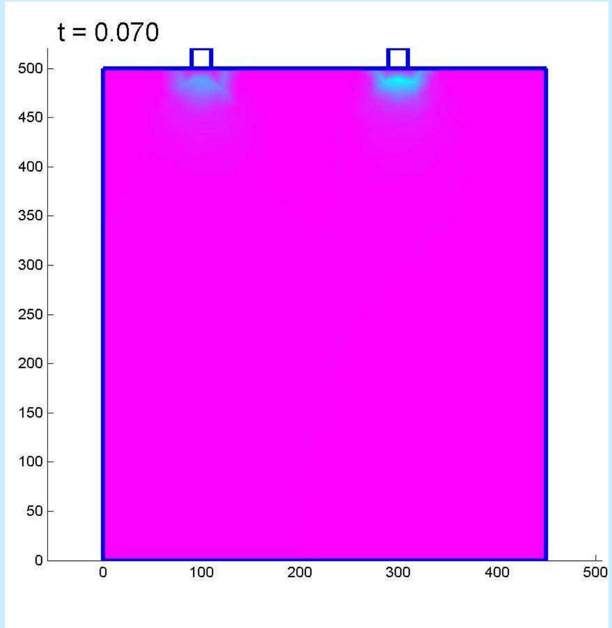
Fracture energy $\gamma = 25 \cdot 10^{-3}$ N/mm

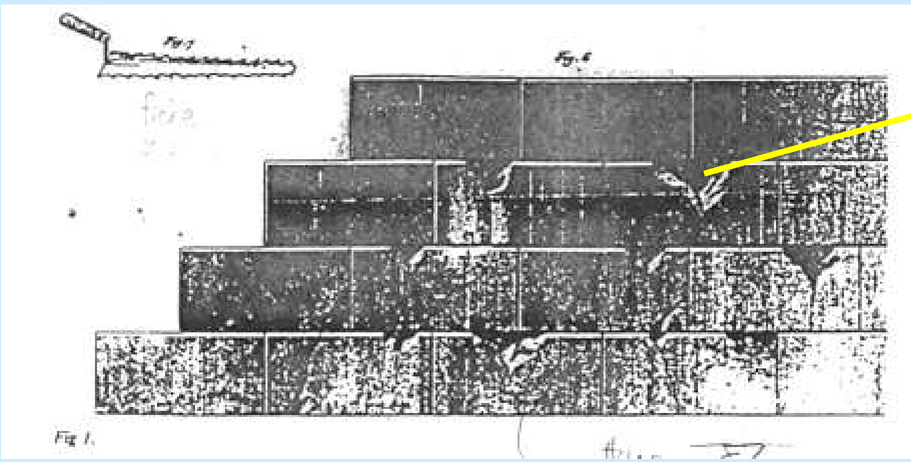
Crack path does not square with that observed

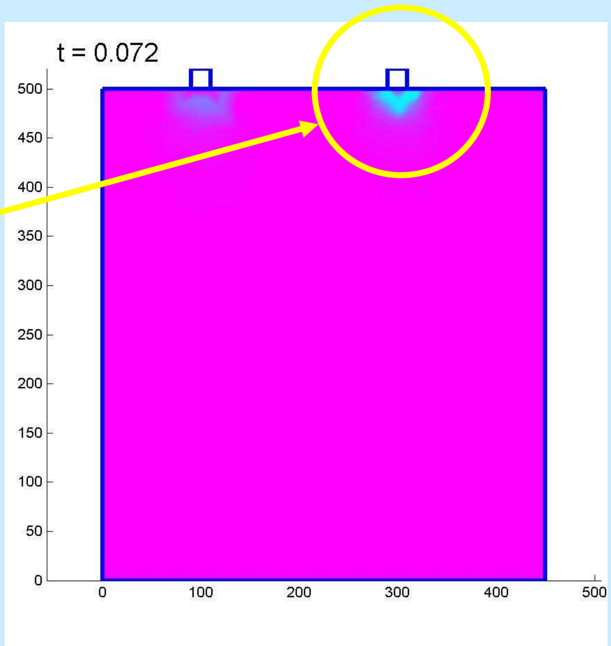
Nevertheless, **OXIDATION CAN PRODUCE DAMAGE**

Crack evolution under contact

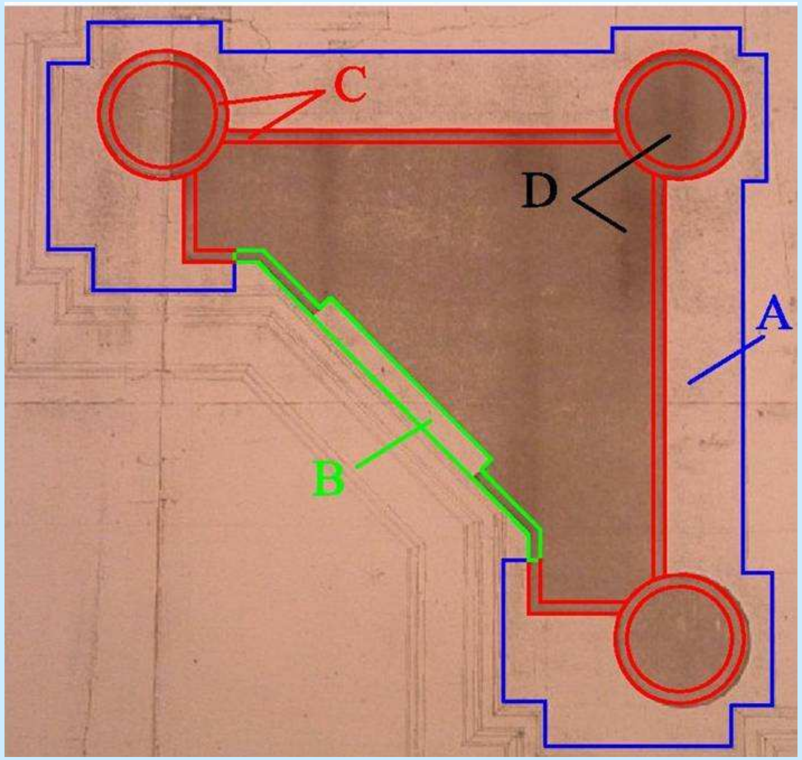


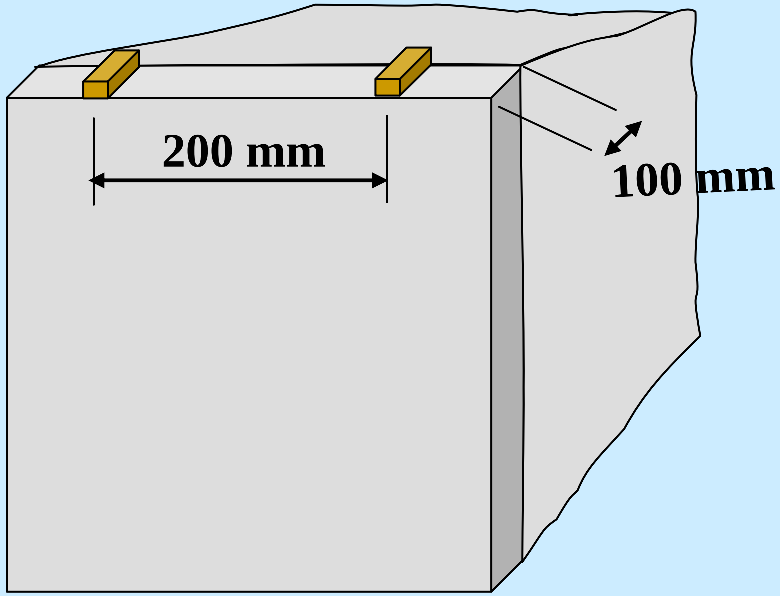






Quantitative results





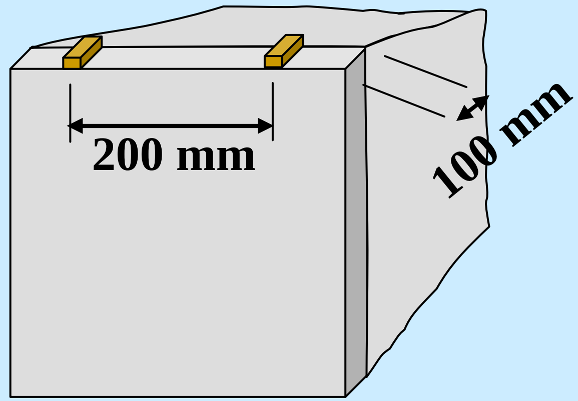
Only the pylon periphery carry the load (length 23 m)

Stress concentrations due to wood wedges

Quantitative results

PEAK LOAD (per unit panel thickness): **350 N/mm**

Displacement t = 0.07 mm



Load on each pylon 34000 kN

Only the pylon periphery carry the load (length 23 m)

Load per unit length = 1480 kN/m

On each wedge (0.2m spaced) F = 1480 kN/m x 0.2 m =300 kN

Wedge 100 mm long ⇒ Pressure = 300000 N / 100 mm = 3000 N/mm

Fracture energy

$\gamma = 25 \cdot 10^{-3}$ N/mm

3000 N/mm >> 350 N/mm ⇒ problems even before dome completion

Conclusions

- Approximations in the model: 2D (not 3D); boundary cond. (heavy use of symmetry)
- Observed crack path equal to cramp pull out simulation
- Iron oxidation cannot reproduce observed crack path
- Pylons may fail during construction (as documented) due to stress concentration from oak wedges
- Iron oxidation may be dangerous
- Revisited model is more efficient than F.B.M. model (mode II fracture and microrotations)


Grazie per l’attenzione
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